Industrial Organization Theory: contracting with externalities in markets

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Tilec

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Outline

Lecture 1: Motivating examples

Outline (cont.)

Lecture 2: More IO examples

- How to license intangible property
- How (not) to sell nuclear weapons
- Incentives and discrimination
- Takeover bids, the free rider problem, and the theory of the corporation
- Technology adoption in the presence of network externalities
- Competition and incentives with nonexclusive contracts

Outline (cont.)

Lecture 3: Offer game and beliefs

Segal and Whinston (2003)

Lecture 4: Bidding games and menu auctions

Bernheim and Whinston (1986)

Martimort and Stole (2003)

Outline (cont.)

Lecture 5: Exclusive dealing

Lecture 6: Details of the bargaining environment

Outline (cont.)

Organization of the course I.O. Theory

- This course is taught from October 23 till December 11, 2018
- Tuesday 10:45-12:30 in PZ 002
- On Nov. 20 and Dec. 4 we also have the reading group
- the lecture on Oct 30 is cancelled
- Course grade is determined by a referee report that you write on an existing paper. See the files with the Grade Requirements and Learning Goals for details.
- if you present in the reading group, you get early feedback on the paper that you choose (but you can also choose different papers for reading group and referee report)
- goal of this course is to understand the literature on bargaining with externalities

Organization of the course I.O. Theory (cont.)

- we will go through a number of papers, usually focusing on the I.O. applications in these papers
- we consider the effects of contracting externalities on parties as well as different ways in which the contracting between parties can be modelled.
- there are no formal home work requirements, but it does say sometimes "*check*" in the lectures and it is highly recommended that you then check such claims at home
- If you cannot figure out how it works, ask me in the next lecture!

Reading group papers: suggestions

- Burguet, Roberto. 2017. "Procurement Design with Corruption." American Economic Journal: Microeconomics, 9(2): 315-41: https://www.aeaweb.org/articles?id=10.1257/mic.20150105
- Bin Liu, Jingfeng Lu, 2018, "Pairing provision price and default remedy: optimal two-stage procurement with private R&D efficiency", RAND Journal, https://onlinelibrary-wiley-com.tilburguniversity.idm.o
- Doh-Shin Jeon, Yassine Lefouili, 2018, "Cross-licensing and competition", RAND Journal, https://doi-org.tilburguniversity.idm.oclc.org/10.1111/

Reading group papers: suggestions (2)

- Hanming Fang and Zenan Wu, 2018, "Multidimensional private information, market structure, and insurance markets", RAND Journal, https://doi-org.tilburguniversity.idm.oclc.org/10.1111/1
- Gabrielsen, Tommy Staahl, and Bjørn Olav Johansen. 2017. "Resale Price Maintenance with Secret Contracts and Retail Service Externalities." American Economic Journal: Microeconomics, 9(1): 63-87. https://www.aeaweb.org/articles?id=10.1257/mic.20140280

Part I

Motivating examples

Coase Theorem (Milgrom and Roberts, 1992, page 38)

If the parties bargain to an efficient agreement (for themselves) and if their preferences display no wealth effects, then the value-creating activities that they will agree upon do not depend on the bargaining power of the parties or on what assets each owned when the bargaining began. Rather, efficiency alone determines the activity choice. The other factors can affect only decisions about how the costs and the benefits are to be shared.

Why don't we always get efficiency?

- *external effects with outsiders:*
  - some people affected by a contract are not present when the contract is bargained on
  - their preferences are not taken into account
  - hence the outcome will not be overall (Pareto) efficient
- *asymmetric information:*
  - see the lecture on Mechanism Design: http://janboone.github.io/RM/LectureMechanismDesign.html
  - suppose downstream retailer R buys from upstream manufacturer M
  - R values quantity/quality of input q at p per unit
  - M produces q at costs $\theta c(q)$ with $\theta \in [0, 1]$ with density (distribution) function f (F), $c' > 0, c'' \geq 0$
  - First, consider the case where R makes take-it-or-leave-it offer

Why don't we always get efficiency? (cont.)

- R offers M a menu of choices $q, t(q)$, such that M's payoffs equal

$$\pi(\theta) = \max_q t(q) - \theta c(q)$$

- Hence it follows that

$$\pi'(\theta) = -c(q(\theta)) < 0 \tag{1}$$

- there is no reason to give rents away and hence (assuming all θ sell $q(\theta) > 0$):

$$\pi(\theta) = \int_\theta^1 c(q(t))dt$$

- Since $t(q(\theta)) = \pi(\theta) + \theta c(q(\theta))$, R solves:

$$\max_{q(.)} \int_0^1 (pq(\theta) - \int_\theta^1 c(q(t))dt - \theta c(q(\theta)))f(\theta)d\theta$$

Why don't we always get efficiency? (cont.)

- using partial integration, this can be written as

$$\max_{q(.)} \int_0^1 (pq(\theta) - \left(\theta + \frac{F(\theta)}{f(\theta)}\right) c(q(\theta)))f(\theta)d\theta$$

- Hence R chooses $q(\theta)$ which solves

$$p - \left(\theta + \frac{F(\theta)}{f(\theta)}\right) c'(q(\theta)) = 0$$

- while efficiency would require $p - \theta c'(q(\theta)) = 0$
- Now, consider the case where M makes take-it-or-leave-it offer.
- *Check* that the outcome is efficient in this case
- Hence in contrast to Coase theorem, it matters who makes the offer

- *hold up:*

Why don't we always get efficiency? (cont.)

- party R cannot commit to abstain from renegotiating the division of surplus later on
- party M can make relation specific investment x to increase value of relationship with R to $V(x)$, with $V'(x) > 0, V''(x) < 0$
- in the next period, surplus is divided by bargaining with bargaining power β for M and $1 - \beta$ for R
- Hence M solves: $\max_x \beta V(x) - x$
- leads to underinvestment since efficient investment solves $\max_x V(x) - x$
- Moreover, in contrast to Coase, bargaining power β affects the efficiency of the outcome

This course: effects of contracting externalities

- Even if all relevant parties are present at the bargaining stage and there is symmetric information, still inefficiencies can arise
- bilateral contracting:
  - introduce inefficiencies to worsen bargaining partners' outside option; you get more rents
  - contracts are privately (not publicly) observed
  - hence contracts need to be "bilaterally stable" between contracting parties; this leads to inefficiencies
- in each model, players need to decide on trades or allocations
- the outcome depends on the details of the bargaining situation (in contrast to Coase)
- However, some outcomes turn out to be robust in the sense that they are predicted by a range of models

What are we interested in?

- how do people cope with these externalities in the real world?
- what are the consequences for welfare?
- how can we model such externalities?
- how does the bargaining structure affect outcomes? Things like: who makes the offer?
- To get some intuition: first consider some examples

Chicago school: exclusive dealing does not lead to foreclosure

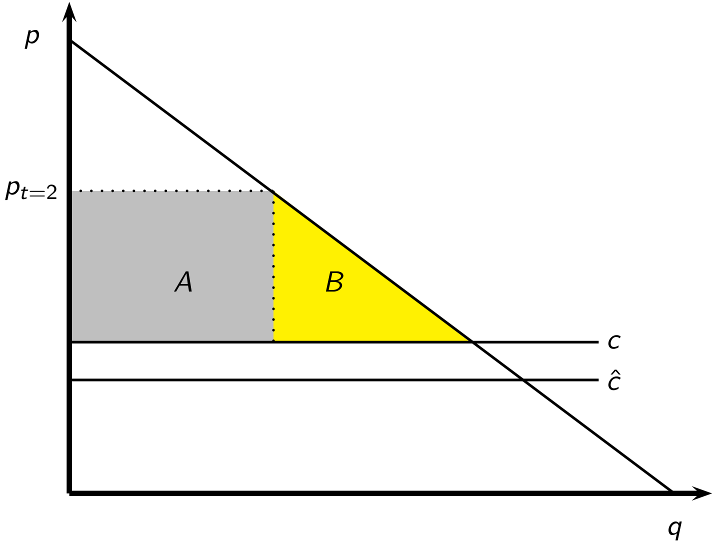
- Assume there is an upstream firm that offers a downstream firm an exclusive dealing contract:
  - the downstream firm can only buy inputs from this upstream firm and not from its competitors
- people used to worry that such contracts are used by the upstream firm to foreclose competitors and hence reduce welfare
- The Chicago school then came up with the argument below to show that exclusive dealing to foreclose competition cannot happen in equilibrium
- Hence the Chicago school argued: if you do see exclusive dealing in reality, it must be because it increases efficiency

Chicago school: exclusive dealing does not lead to foreclosure (cont.)

- The Chicago argument has been subsequently attacked by models that introduce contracting externalities
- Consider situation where at $t = 1$ there is one incumbent U and one D
- In $t = 2$ an entrant $\hat{U}$ may appear in the upstream market who is more efficient than U: $\hat{c} < c$
- Assume U and $\hat{U}$ sell a homogenous product and compete in prices
- To avoid losing business in the next period, U offers D this period an exclusive dealing contract
- If D accepts, $\hat{U}$ cannot enter as it has no downstream firm (*e.g.* retailer) to sell to

Chicago school: exclusive dealing does not lead to foreclosure (cont.)

- Will D accept such a contract?
- Chicago School: No


p
$p_{t=2}$
A
B
c
$\hat{c}$
q

No foreclosure in equilibrium

- if D does not sign the contract, Bertrand competition leads to $p = c$ in $t = 2$
- because of exclusivity contract, U can sell to D at a price $p_{t=2} > c$
- gain to U of doing this equals area A
- loss to D of the exclusivity contract equals areas $A + B$
- Hence U can never profitably compensate D for accepting the exclusivity contract and hence D should not accept such a contract
- Chicago School: if you see such a contract in reality, it must be that it creates efficiency gains and hence it is welfare enhancing
- Policy implication: no intervention required against exclusive dealing contracts: inefficient foreclosure cannot happen

Damage clauses

- externality on entrant who is not present
- U and $\hat{U}$ offer surplus $S = 1$ to D (one downstream firm)
- U has cost $c = \frac{1}{2}$; $\hat{U}$ has cost $\hat{c}$ uniformly distributed on $[0, 1]$
- U offers D a contract that says D buys from U at price p and otherwise pays penalty d
- Hence D only switches to $\hat{U}$ if $\hat{p} + d < p$
- without contract, two situations
  - $\hat{c} < \frac{1}{2}$, $\hat{U}$ enters and charges $\hat{p} = \frac{1}{2}$, $\Pi_{\hat{U}} = c - \hat{c}, \Pi_U = 0, \Pi_D = S - c$
  - $\hat{c} \geq \frac{1}{2}$, $\hat{U}$ does not enter $p = S$, $\Pi_{\hat{U}} = 0, \Pi_U = S - c, \Pi_D = 0$
  - Expected profit U equals $\frac{1}{2}(S - c) = \frac{1}{4}$ and D gets $\frac{1}{2}0 + \frac{1}{2}(1 - \frac{1}{2}) = \frac{1}{4}$

Damage clauses (cont.)

- Now U offers D a contract $(p = 3/4, d = \frac{1}{2})$; D is willing to accept because
  - pay off D equals $Pr(\hat{c} \geq p - d)(1 - p) + Pr(\hat{c} < p - d)(1 - p) = \frac{1}{4}$
  - and U gains as $Pr(\hat{c} \geq p - d)(p - c) + Pr(\hat{c} < p - d)(d) = 5/16 > \frac{1}{4}$
  - Contract is signed in equilibrium but welfare reducing
- contract between U and D is profitable (though inefficient) because of negative externality on $\hat{U}$ who is not present when bargaining takes place
- See BW98 below for the case where $\hat{U}$ can make offers as well

Naked exclusion

- Now consider the situation in which we have two downstream firms D_1, D_2 and one upstream firm U
- U offers exclusive dealing contracts to these downstream firms
- if D_i accepts U's exclusive dealing contract, he cannot buy from an entrant
- the entrant can only come into the market if she can sell to both downstream firms
- Hence U only needs one downstream firm to accept the contract to keep the entrant out
- U offers D_i a payment x_i in order to accept the contract
- if one or both downstream firms accept the contract, they have to pay U the monopoly price; downstream profits for each firm equal 165

Naked exclusion (cont.)

- without entry U earns the monopoly profit equal to 500
- if entry occurs, competition between entrant and incumbent leads to lower input prices and each downstream firm has a profit equal to 500
- in case of entry monopolist earns his outside option: 50

decision of D_2

decision of D_1		Accept	Reject
	Accept	$165 + x_1$, $165 + x_2$	$165 + x_1$, 165
	Reject	165, $165 + x_2$	500, 500

Table: Payoffs of downstream firms

Naked exclusion (cont.)

- first note that there is an externality on the entrant (who is not present)
- moreover there are externalities among parties involved in bargaining because contracts are bilateral: x_i cannot depend on whether $j \neq i$ accepted U's contract
- hence the outcome can be inefficient even if we consider the payoffs of U, D_1, D_2
- we consider 4 different bargaining situations:
- *non-discriminatory* offers: U has to offer each D_i the same $x_1 = x_2 = x$:
    - to compensate D_i to accept the contract conditional on D_{-i} rejecting it, requires $x = 500 - 165 = 335$
    - hence if $x < 335$ and D_i rejects the offer, it is optimal for D_{-i} to reject the offer as well

Naked exclusion (cont.)

- further note, that if $x > 0$ and D_i accepts, it is optimal for D_{-i} to accept as well
- however, if both accept $x = 335$, U earns $500 - 2 * 335 < 50$
- Thus offering $x = 335$ and both accepting cannot be an equilibrium
- *check* that any $x \in [0, 225]$ and both accepting is a Nash equilibrium
- any $x \in [0, 335]$ and both rejecting is a Nash equilibrium as well
- if D_1, D_2 can coordinate, they would prefer to reject offers $x < 335$

- *public discriminatory* offers: U offers D_1, D_2 simultaneously offers x_1, x_2 which can be different:
  - now U can force D_1 to accept by offering $x_1 = 335$ and $x_2 = 0$ because $500 - 335 > 50$

Naked exclusion (cont.)

  - outcomes with $x_1 + x_2 \in [0, 335]$ and both downstream firms accept can also be sustained as equilibria

- *private discriminatory* offers: U offers D_1, D_2 simultaneously x_1, x_2 but D_i does not observe the offer to D_{-i}:
  - here we need to specify *beliefs*:
    - suppose U's equilibrium offers are $\hat{x}_1, \hat{x}_2$
    - to check whether this is an equilibrium, we need to verify what happens to U's payoffs if U offers $x_i \neq \hat{x}_i$ to D_i
    - if D_i receives offer $x_i \neq \hat{x}_i$, what does he believe about x_{-i}?
  - this literature tends to impose *passive beliefs*: if D_i receives $x_i \neq \hat{x}_i$ he believes that D_{-i} received $\hat{x}_{-i}$
  - unique Perfect Bayesian Nash equilibrium is $x_1 = x_2 = 0$ and both downstream firms accept the offer:
    - an offer (x_1, x_2) that is rejected by both downstream firms cannot be an equilibrium as U can profitably deviate to an offer $x_1' = 335$ that is optimal to accept for D_1

Naked exclusion (cont.)

- consider an offer (x_1, x_2) with $x_2 \in [1, 335]$: if D_1 accepts, D_2 should accept as well in equilibrium but then U could have saved money by setting $x_2 = 0$; if D_1 rejects, D_2 should reject as well; but this cannot be an equilibrium because of the first point
- this reasoning is correct for any $x_1 < 335$ and hence it is true for $x_1 = 0$
- hence $x_1 = x_2 = 0$ and both downstream firms accept is the only equilibrium under passive beliefs.

- *sequential* offers: U first makes an offer x_1 to D_1, after D_1 has decided whether to accept/reject this offer and D_2 has observed this decision, U makes an offer x_2 to D_2:
  - in the Subgame Perfect Nash equilibrium, U offers $x_1 = \varepsilon > 0$ but small to D_1 who accepts, then U offers $x_2 = 0$ to D_2 who can either accept or reject

Naked exclusion (cont.)

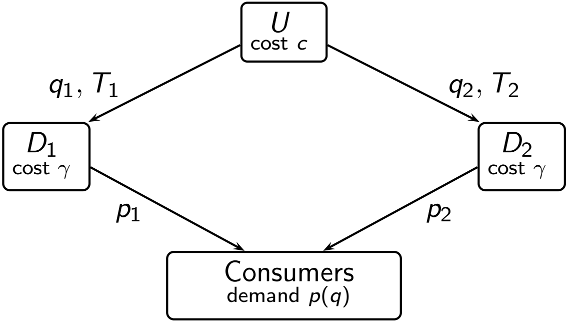
- to see why this is an equilibrium: suppose D_1 rejects $x_1 = \varepsilon > 0$, then the optimal offer for U is $x_2 = 335$ who accepts; hence this is an unprofitable deviation for D_1

Private offers

- above we considered the situation where U considers exclusive dealing contracts because of potential entry by another upstream firm
- HT90 consider the problem where U sells to two downstream firms without the danger of entry by another upstream firm
- U now offers a downstream firm D_i an exclusive deal that implies that U only sells to D_i and not to D_{-i}
- the fear is that U tries to leverage its market power (monopoly) in the upstream market into the downstream market which is more competitive (duopoly)
- Chicago School: in this situation exclusive dealing cannot be welfare reducing
    - U can get monopoly profit on final good market anyway

Private offers (cont.)

  - hence exclusive dealing must have an efficiency rationale
- HT90 assume that U cannot commit to a given output level
- put differently, U makes private offers to downstream firms
- If there are two downstream firms D_1, D_2, Chicago school predicts that U sells $q_i = q^m/2$ to each of them and charges each D_i: $T_i = \frac{1}{2}(p^m q^m - (c+\gamma)q^m)$
- where c is U's constant marginal cost, γ is D_i's constant marginal cost and q^m, p^m denote the monopoly output, price resp.


U
cost c
q_1, T_1
q_2, T_2
D_1
cost γ
D_2
cost γ
p_1
p_2
Consumers
demand $p(q)$

Exclusive contract to restore market power

- If D_i cannot observe what contract D_j gets, is she still willing to pay $T_i = \frac{1}{2}(p^m q^m - (c+\gamma)q^m)$ for $q_i = q^m/2$?
- Given that i bought q_i, she should expect U to sell q_j to j which solves $\max_q\{p(q_i+q)q - (c+\gamma)q\}$
- Hence, given q_i, the output level q_j solves

$$p'(q_i+q_j)q_j + p(q_i+q_j) - (c+\gamma) = 0$$

- In a symmetric equilibrium this is the Cournot output: $q^C > q^m/2$
- the Cournot outcome is *bilaterally stable*
- In contrast to the Chicago School's argument, U does not get monopoly profit but less because $q^C + q^C > q^m$

Exclusive contract to restore market power (cont.)

- problem is that U cannot contract with D_i on q_{-i}
- however, suppose that U can contract with D_1 upon whether $q_2 = 0$: i.e. U gives D_1 an exclusive dealing contract.
- this allows U to reap the monopoly profit
- such an exclusive dealing contract raises U's profits but reduces welfare

Menu auctions in auction context

- Suppose principal has 2 objects a, b which she values at 0
- two bidders 1,2 with valuations for the objects:

allocation	u_1	u_2
Nothing	0	0
x_a	6	5
x_b	5	6
x_a and x_b	8	7

- goods are partial substitutes:
  $u_i(x_a) + u_i(x_b) > u_i(x_a + x_b) > u_i(x_a), u_i(x_b)$
- bidders 1,2 simultaneously make bids t_1, t_2 for different allocations, bidders pay their winning bid (first price auction)
- efficient allocation is: 1 gets a and 2 gets b; total surplus 12
- $U_i = u_i - t_i$ and $U_p = t_1 + t_2$

Menu auctions in auction context (cont.)

- principal chooses allocation that maximizes U_p
- inefficient equilibria exist: $t_1(x_a + x_b) = t_2(x_a + x_b) = 7$ and $t_1(x_a) = t_1(x_b) = t_2(x_a) = t_2(x_b) = 0$
- *check* that this is an equilibrium
- leads to allocation where 1 gets a and b; total surplus equals $8 < 12$
- efficient equilibrium: $t_1(x_a) = 5, t_1(x_b) = 0, t_2(x_a) = 0, t_2(x_b) = 2$ and $t_i(x_a + x_b) = 7$ for $i = 1, 2$, 1 gets a, 2 gets b
- what is maximum surplus that 1 and 2 can get in an efficient equilibrium?
  - Let $S^* = u_1(x_a) + u_2(x_b) = 12$
  - coalition of P and 2 get in equilibrium $S^* - U_1$

Menu auctions in auction context (cont.)

- if they deviate and stop trading with 1, they can get $u_2(x_a + x_b) = 7$
- Hence $S^* - U_1 \geq 7$ or equiv. $U_1 \leq 5$
- similarly $U_2 \leq 4$

- restrict attention to bids $t_i(.)$ such that $t_i(x) = u_i(x) - \bar{u}_i$ where x denotes a matrix with $x_{ij} = 1$ if player i gets object j etc.
- such bids are called *truthful* because they truthfully reveal a bidder's marginal willingness to pay:

$$t_i(x) - t_i(y) = u_i(x) - u_i(y) \tag{2}$$

- $\bar{u}_i$ denotes the utility that i gets when it wins
- Note that in any equilibrium $t_1(x_a + x_b) = t_2(x_a + x_b)$:

Menu auctions in auction context (cont.)

- suppose not, i.e. suppose $t_1(x_a + x_b) < t_2(x_a + x_b)$. Two cases:
  1. 2 gets a and b: he can reduce $t_2(x_a + x_b)$ (and, if necessary $t_2(x_a), t_2(x_b)$) and still get a and b but now at lower price;
  2. 2 gets only one good at price $t_2(x_j)$: then it must be the case that $t_1(x_i) + t_2(x_j) = t_2(x_a + x_b)$ (what is the contradiction if either $t_1(x_i) + t_2(x_j) < t_2(x_a + x_b)$ or $t_1(x_i) + t_2(x_j) > t_2(x_a + x_b)$?), hence 2 can reduce $t_2(x_j)$ and $t_2(x_a + x_b)$ and increase payoffs

- Further, P allocates a to 1 and b to 2:
- with truthful bids, P chooses x to solve

$$\max_x t_1(x) + t_2(x) = u_1(x) + u_2(x) - \bar{u}_1 - \bar{u}_2 = S^* - \bar{u}_1 - \bar{u}_2$$

- Hence we have
$t_1(x_a) + t_2(x_b) = t_1(x_a + x_b) = t_2(x_a + x_b) \equiv t(x_a + x_b)$:

Menu auctions in auction context (cont.)

- what is the contradiction if either $t_1(x_a) + t_2(x_b) < t_1(x_a + x_b)$ or $t_1(x_a) + t_2(x_b) > t_1(x_a + x_b)$?

- *check* that using truthful bids and equation (2), we can solve

$$t(x_a + x_b) = t(x_a + x_b) - (u_1(x_a + x_b) - u_1(x_a)) + \\ + t(x_a + x_b) - (u_2(x_a + x_b) - u_2(x_b)) \quad (3)$$

- hence we find: $t(x_a + x_b) = (u_1(x_a + x_b) - u_1(x_a)) + (u_2(x_a + x_b) - u_2(x_b)) = 2 + 1 = 3$
- consequently, $t_1(x_a) = 1, t_2(x_b) = 2$ and $U_1 = 6 - 1 = 5$, $U_2 = 6 - 2 = 4$ which equal the upper-bounds derived above
- Hence menu auctions lead to efficient outcomes if players use truthful bids
- players then receive their marginal contribution to the surplus

Concepts

- For most of the lectures: one party (monopolist) on one side of the market and two (or more) parties on the other side of the market
- bilateral bargaining: outcome should be such that there is no incentive for two contracting parties to deviate
- party making the offers:
  - bidding game: side of the market with 2 or more parties makes the offers (common agency)
  - offer game: side of the market with monopolist makes the offers
- public/private contracts:
  - passive beliefs
  - symmetric beliefs

Concepts (cont.)

- simple (singleton) contracts
- menu of contracts (with symmetric information about players' types)
- Observable and verifiable (i.e. contractible):
  - x_i, x_{-i}
  - only x_i (private offers)
  - x_i and whether $x_{-i} = 0$
- (non)discrimination
- simple/unique implementation
- externalities on traders/non-traders
- positive/negative externalities
- increasing/decreasing externalities

Part II

More IO examples

How to license intangible property

How (not) to sell nuclear weapons

Incentives and discrimination

Takeover bids, the free rider problem, and the theory of the corporation

Technology adoption in the presence of network externalities

Competition and incentives with nonexclusive contracts

Katz and Shapiro (1986a)

- An upstream firm/research lab has a patent which can reduce the marginal cost of production for *n* symmetric downstream firms
- externalities:
  - the profits of a firm that does not license from the upstream firm are affected by the number of downstream firms that do get a license
  - the profits of a firm that does license from the upstream firm are affected by the number of downstream firms that do get a license
  - here only the number of firms that license matter (not their identity; see Jehiel et al. (1996) below)
- assume that license fee can only take the form of a fixed fee (i.e. not a royalty rate per unit of output sold)

Katz and Shapiro (1986a) (cont.)

- questions:
  - how to maximize revenue from the patent?
  - should the lab sell to all downstream firms?
  - does lab sell the socially efficient number of licenses?
  - does it matter whether the lab is owned by $m \leq n$ downstream firms (research joint venture)?

Model

- Let k denote the number of firms that get a license
- Let $W(k)$ denote the profits of a firm with a license when there are $k-1$ other firms with a license
- Let $L(k)$ denote the profits of a firm without a license when there are k firms with a license
- Assume that $L(k) < W(k)$

$$L(k) \leq L(k-1) \tag{4}$$

- profits with a license are higher than without a license
- if you don't have a license, your profits fall with the number of firms with a license

Model (cont.)

- Let $(k, \underline{b})$ denote a k unit sealed-bid auction with reserve price $\underline{b} \geq 0$
  - firm can only bid for one license
  - highest bidder receives first unit and pays his bid, if $k \geq 2$ next highest bidder receives second unit etc.
- i's willingness to pay is given by $\bar{b}_i = W(k^i) - L(k^{-i})$ where
  - k^i denotes number of producers (including i) that buy a license
  - k^{-i} number of producers that buy a license if i does not (two cases: see below)
- In any equilibrium all firms that buy a license pay the same price (*check*: why?)
- we consider two cases: upstream firm is (i) an independent lab and (ii) research joint venture owned (equally) by m downstream firms

Independent lab

- quantity restriction k affects expectations about k^{-i}:
  - if $k < n$ then $k^{-i} = k$
  - if $k = n$ then $k^{-i} = k - 1$
- bids and reserve price:
  - if $k < n$, seller can use mechanism $(k, 0)$ and each firm bids $W(k) - L(k)$
  - if $k = n$, seller uses $(n, \underline{b})$ with reserve price $\underline{b} = W(n) - L(n-1)$
- define $V(k) \equiv W(k) - L(k-1)$
- two cases:
  - $V'(k) < 0$, with $(n, \underline{b})$ lab can earn $nV(n)$
  - $V'(k) \geq 0$

Independent lab (cont.)

- this can happen if the innovation establishes a new industry standard: W(k+1) > W(k)
- then we have

$$V(k+1) - V(k) = W(k+1) - W(k) + L(k-1) - L(k) > 0$$

because of equation (4)

- if seller uses $(n, \underline{b})$, there can exist multiple bidding equilibria:
- equilibrium with $k = n$ exists if $V(n) \geq \underline{b}$
- equilibrium with $k = 0$ exists (as well) if $V(1) \leq \underline{b}$
- if both equilibria exist and $W(n) - \underline{b} > L(0)$ then all downstream firms prefer equilibrium with $k = n$
- one way to break the equilibrium with $k = 0$ is for the seller to do the following: if the number of bidders is less than or equal to $n - 1$, each bidder gets the license for free; if there are n bidders, each pays $V(n)$ for license; *check* that in equilibrium lab earns $nV(n)$

Independent lab (cont.)

- other way to break the $k = 0$ equilibrium is to make discriminating offers to downstream firms (see below: Winter (2004))

- Let $R^0(k)$ denote the profits that an independent lab can earn by selling k licenses, then we have shown that

Independent lab (cont.)

Proposition 1

An independent research lab's selling strategy takes one of the following forms:

(a) $(k, 0)$ *with* $k < n$, *winning bid equals* $W(k) - L(k)$ *and*

$$R^0(k) = k(W(k) - L(k))$$

(b) $(n, \underline{b})$ *with* $\underline{b} = W(n) - L(n-1)$ *and*

$$R^0(n) = nV(n)$$

Research joint venture

- the research lab is owned equally by $m \leq n$ downstream firms (insiders)
- venture issues k licenses, $\tilde{k} \leq m$ to insiders and receives revenue R from outsiders
- total insider profits equal

$$R^m(k) = \tilde{k}W(k) + (m - \tilde{k})L(k) + R$$

- if $k < n$ then we know from above that $R = (k - \tilde{k})(W(k) - L(k))$ and hence

$$R^m(k) = k(W(k) - L(k)) + mL(k) = R^0(k) + mL(k)$$

- note that this does not depend on $\tilde{k}$

Research joint venture (cont.)

- with $(n, \underline{b})$ we have

$$\begin{aligned} R^m(n) &= mW(n) + (n-m)(W(n) - L(n-1)) \\ &= nV(n) + mL(n-1) \\ &= R^0(n) + mL(n-1) \end{aligned}$$

- hence the *m*-firm joint venture has same selling strategies as in proposition 1; expression for $R^m(k)$ has additional $mL(k)$ term compared to $R^0(k)$

Optimal number of licenses

- let k^m maximize $R^m(k)$ (with $m = 0$: indep. lab)
- then k^n maximizes industry profits
- two cases:

$k < n$ *check* that

$$\begin{aligned}\Delta R^m(k) &= R^m(k) - R^m(k-1) \\ &= (R^0(k) - R^0(k-1)) \\ &+ m(L(k) - L(k-1))\end{aligned}$$

$k = n$ *check* that

$$\Delta R^m(n) = \Delta R^0(n)$$

Optimal number of licenses (cont.)

- hence ΔR^m is weakly decreasing in m

Corollary 1

Any venture with $m \leq n-1$ sells (weakly) too many licenses from the point of view of maximizing industry profits

- as we will learn later on (Segal (1999)) this is caused by a negative externality on non-traders

Optimal auction to sell licenses

- above we assumed that only firms that buy a license pay a fee to the lab
- however, the lab could charge a license fee E to participate in the mechanism/auction
- Two cases (recall k^n maximizes $R^n(k)$):

  $k^n < n$ $E = L(k^n) - L(n-1)$: lab announces that all n firms have to pay E; if fewer than n firms actually pay E, each of these paying firms gets E reimbursed and gets the license for free; if all n firms pay E, licenses are auctioned using $(k^n, 0)$

Optimal auction to sell licenses (cont.)

$k^n = n$ $E = W(n) - L(n-1)$: all n firms have to pay E, if all of them do they participate in $(n, 0)$; if not all n firms pay E, paying firms get E reimbursed and the innovation for free

- to see why this works:
  - if $n-2$ or fewer firms pay E, one of the remaining firms has an incentive to pay E as well (gets E reimbursed and the innovation for free); hence cannot be an equilibrium
  - if $n-1$ firms pay E, remaining firm earns $L(n-1)$ if it does not pay E and it earns $L(n-1)$ if is does pay E
- hence there exists an equilibrium where all n firms pay E
- in the equilibrium with $k^n < n$ all firms pay E but not all of them get a license: these firms pay to avoid $k > k^n$
- why is this optimal for the lab?

Optimal auction to sell licenses (cont.)

- with $k = k^n$ downstream industry profits are maximized
- each outsider gets payoff $L(n-1)$ which is as low as possible (see equation (4))
- hence revenue for the lab is maximized
- it is important that the lab can commit to this mechanism: if fewer than n firms pay E, lab earns 0

Incentive to innovate

- social value of innovation equals (where we use $W(0) = L(0)$):

$$kW(k) + (n-k)L(k) - nL(0)$$

- private value (using optimal mechanism with $k < n$) for a joint venture with m insiders equals

$$(n-m)E + (k-\tilde{k})(W(k) - L(k)) + \tilde{k}W(k) + (m-\tilde{k})L(k)$$
$$-mL(0) =$$
$$kW(k) + (n-k)L(k) - nL(0) - (n-m)(L(n-1) - L(0))$$

- because $L(n-1) < L(0)$, private incentive to innovate is excessive for lab with $m < n$

Incentive to innovate (cont.)

- lab can extract license payments that exceed the change in industry profits
- *check* that this result also holds with $k = n$

Jehiel et al. (1996)

- Katz and Shapiro (1986a) payoffs only depend on the number of licenses sold, not on the identity of the buyer
- here we analyze the optimal mechanism for the case where externalities are firm dependent
- seller wants to sell one unit of an indivisible good
- if buyer $i \in B = \{1, ..., n\}$ buys the good at price p, i's payoff equals $\pi_i - p$
- payoff to buyer j if i gets the good: $-\alpha_{ij} \leq 0$
- if seller (buyer 0) keeps the good, utilities of all agents normalized to 0: $\alpha_{0j} = 0$ for all j
- let $\alpha^i = \max_{j \neq i} \alpha_{ji}$ denote i's worst outcome if the good is sold but not to i

Jehiel et al. (1996) (cont.)

- $v(i) = \{h | \alpha_{hi} = \alpha^i\}$: set of players j that (when one of them buys) leads to i's worst outcome
- to be able to break ties; let ε denote the smallest money unit
- before we analyze the optimal mechanism, what is the maximum revenue R^* that the seller can get?
- two cases:
  - seller does not sell; i will not pay more than α^i in this case and hence

$$R^* \leq \sum_{i=1}^{n} \alpha^i$$

Jehiel et al. (1996) (cont.)

- seller sells to i: i does not pay more than $\pi_i + \alpha^i$; $j \neq i$ does not pay more than $\alpha^j - \alpha_{ij}$ and hence

$$R^* \leq \pi_i + \alpha^i + \sum_{j \neq i} (\alpha^j - \alpha_{ij})$$

- consider the following mechanism based on the set B^* of buyers that participate:
  - if $B^* = \emptyset$ then the seller keeps the object
  - if $|B^*| \leq n - 2$ then seller chooses (arbitrarily) player $i = \min_j \{j \in B^*\}$ as winner; payments are determined as follows:
    - consider a buyer $k \in B^*$ and define $j = \min_l \{l \in B^* \setminus \{k\}\}$
    - if $k = i$ (winner), then k pays $\pi_i + \alpha_{ji} - \varepsilon$ (threat: if i does not participate in the mechanism, j will get the product)
    - if $k \neq i$, k pays $\alpha_{jk} - \alpha_{ik} - \varepsilon = -\varepsilon$

Jehiel et al. (1996) (cont.)

- if $B^* = B \setminus \{h\}$, winner is $v(h)$ (i.e. h's worst outcome)
  - consider $k \in B^*$ and define $j = \min_j B^* \setminus \{k\}$
  - if $k = v(h)$ then k pays $\pi_k + \alpha_{jk} - \varepsilon$
  - if $k \neq v(h)$ then k pays $\alpha_{jk} - \alpha_{v(h)k} - \varepsilon$
- if $B^* = B$, then two cases (depending on comparison of $\sum_{i=1}^n \alpha^i$ and $\pi_i + \alpha^i + \sum_{j \neq i}(\alpha^j - \alpha_{ij})$):
  - $\max_i \pi_i - \sum_{j \neq i} \alpha_{ij} < 0$: seller keeps the good, i is required to pay $\alpha^i - \varepsilon$ (threat: if i does not pay, then $v(i)$ gets the good)

$$R = \sum_i (\alpha^i - \varepsilon)$$

Jehiel et al. (1996) (cont.)

- $\max_i \pi_i - \sum_{j \neq i} \alpha_{ij} \geq 0$: seller sells to $k \in \arg\max_i \pi_i - \sum_{j \neq i} \alpha_{ij}$; k pays $p = \pi_k + \alpha^k - \varepsilon$; $j \neq k$ pays $\alpha^j - \alpha_{kj} - \varepsilon \geq 0$ (by definition of α^j); threat if j does not pay, good is sold to $v(j)$

$$R = \sum_i (\alpha^i - \varepsilon) + \pi_k - \sum_{i \neq k} \alpha_{ki}$$

Proposition 2

Everyone participates in the mechanism described above. Further,

$$R^* \leq R + n\varepsilon \tag{5}$$

Jehiel et al. (1996) (cont.)

Proof

- Equation (5) follows immediately from the expressions for R^* and R. It implies that R converges to R^* as ε goes to zero: mechanism achieves highest possible revenue.
- To check that all buyers participate, we go over the cases:
  - $B^* = B \setminus \{i\}$: if i does not participate, i's payoff equals $-\alpha^i$; *check*: if i does participate, i's payoff equals $-\alpha^i + \varepsilon$; hence participation is optimal
- Hence, there exists an equilibrium in which everyone participates
- In fact, participating is a strictly dominant strategy:
  - $B^* = B \setminus \{i, m\}$: if i does not participate, payoff equals $-\alpha_{ji}$; with participation $-\alpha_{ji} + \varepsilon$
  - *check*: same is true in case $|B^*| \leq n-2$ $\square$

Winter (2004)

- consider a team of n agents that needs to complete a project
- all n agents are symmetric in their abilities and in their contribution to the success of the project
- agents' effort is not observable (not to the principal nor to other agents)
- only success of the project is contractable; agents get reward conditional on success
- principal wants to *guarantee* success at the lowest cost
- intuitively, one expects that symmetric rewards are optimal
- however, because of the positive externality between team members, the optimal mechanism rewards agents differently (discrimination)

Winter (2004) (cont.)

- by promising some agents high rewards, the others know that they will contribute effort for sure; this makes it cheaper to induce the others to invest effort
- trade off between efficiency and equity in a perfectly symmetric set up
- difference with Segal (2003a) is that "trade" (effort) is not contractable
- agent i can either invest effort ($d_i = 1$) at cost c or shirk ($d_i = 0$) at no cost
- probability that agent i's task is completed successfully equals α^{1-d_i} with $\alpha \in \langle 0, 1 \rangle$
- project is successful if all n tasks are completed successfully

Winter (2004) (cont.)

- probability that project is completed successfully equals $\alpha^{n-\sum_{i=1}^{n} d_i}$
- agent i receives reward ν_i if the project is completed successfully and 0 otherwise
- what is the lowest cost at which the principal can guarantee (i.e. unique equilibrium) that the project is completed successfully?
- rank agents such that $\nu_1 \leq \nu_2 \leq ... \leq \nu_n$
- claim: $\nu_i^* = \frac{c}{(1-\alpha)\alpha^{i-1}}, i = 1, ..., n$ (plus ε) guarantees success of the project at the lowest cost
- proof in two steps:

Winter (2004) (cont.)

- each agent i invests effort $d_i = 1$: consider an agent i who knows that agents $\{i+1, ..., n\}$ invest effort, then i invests effort as well because

$$\alpha^{i-1} * 1 \frac{c}{(1-\alpha)\alpha^{i-1}} - c \geq \alpha^i \frac{c}{(1-\alpha)\alpha^{i-1}}$$

- can we guarantee success at a cost lower than $\sum_i \nu_i^*$?
- rank agents such that $\nu_1 \leq \nu_2 \leq ... \leq \nu_n$
  - if the cost is lower than $\sum_i \nu_i^*$, there exists an agent i such that $\nu_i < \frac{c}{(1-\alpha)\alpha^{i-1}}$
  - let r be the highest index such that $\nu_r < \frac{c}{(1-\alpha)\alpha^{r-1}}$
  - then r knows that agents $\{r+1, ..., n\}$ invest effort, but he does not know whether agents $\{1, ..., r-1\}$ invest effort

Winter (2004) (cont.)

- hence there exists an equilibrium in which r does not invest effort because

$$\alpha^{r-1} * 1 \frac{c}{(1-\alpha)\alpha^{r-1}} - c < \alpha^r \frac{c}{(1-\alpha)\alpha^{r-1}}$$

- because ν_i is weakly increasing in i, all players $\{1, ..., r\}$ shirk
- hence it is not possible to guarantee success of the project at a cost below $\sum_i \nu_i^*$

- due to the positive externality that other agents' effort investments increase the return to your effort, optimal rewards involve discrimination in a perfectly symmetric set up
- in fact, no two agents receive the same reward

Grossman and Hart (1980)

- consider a firm with value (under current management): v
- raider can add value by taking over the firm $\hat{v} = v + 1$
- shareholders $[0, 1]$; each owns one share
- raider needs $\kappa \in \langle 0, 1]$ shares to get control of the firm
- suppose raider offers to buy each share tendered to her at price $v + P$: $P \in [0, 1]$ is premium over current price
- let β denote the probability that take over succeeds
- Then $P = \beta$; to see this note that shareholder gets
    - $v + P$ in case he tenders
    - $(1 - \beta)v + \beta(v + 1) = v + \beta$ in case he does not tender
- Proof by contradiction. Suppose that $P \neq \beta$:

Grossman and Hart (1980) (cont.)

- if $\beta > P$, then no one tenders and $\beta = 0 \leq P$: contradiction
- if $\beta < P$, then everyone tenders and $\beta = 1 \geq P$: contradiction

- equilibrium where κ shares are traded with probability 1; payoff raider equals

$$\pi = \kappa(v + 1 - (v + P)) = 0$$

- raider has no payoff: all the surplus that the raider can create is transfered to shareholders
- unlikely that raider will try to takeover firm: social value $\hat{v} - v > 0$ is lost
- To see why this happens, consider a bid by the raider with $P \in \langle 0, \beta \rangle$:

Grossman and Hart (1980) (cont.)

- in equilibrium no one tenders and hence payoff to shareholders equals v
- Segal (1999): with positive externalities on non-traders, trade is inefficiently low in equilibrium
- if κ shares were tendered, shareholders that tender earn $v + P > v$; shareholders that didn't tender earn $v + \beta > v + P$
- all shareholders would gain from this
- however, tendering is a public good: not tendering yields higher payoffs than tendering
- no shareholder is pivotal and all of them free ride
- positive externality of tendering

- possible solution for the raider: make a conditional offer on all shares at premium $P = \varepsilon > 0$ arbitrarily small
- that is, raider only buys shares if she can buy all shares

Grossman and Hart (1980) (cont.)

- each shareholder is pivotal now:
  - if shareholder does not tender, payoff equals v
  - if shareholder does tender, payoff equals $v + \varepsilon > v$.
- everyone tenders and the raider's payoff is positive $(1 - \varepsilon > 0)$

Katz and Shapiro (1986b)

- We consider the dynamics of industry evolution in a market with network externalities
- two time periods $t = 1, 2$
- N_t number of consumers at time t
- $x_t(y_t)$ number of consumers in period t that buy technology $A(B)$
- $x_t + y_t = N_t$
- utility of agent at time t who buys A at price p_t

$$v(x_1 + x_2) - p_t$$

and similarly for B at price q_t:

$$v(y_1 + y_2) - q_t$$

Katz and Shapiro (1986b) (cont.)

- hence network effect is determined by the total number of users at time 2
- consumers buying at $t = 1$ need to predict x_2, y_2 (assume rational expectations)
- we assume that A is better than B in both periods: i.e. constant marginal costs are lower for A than for B in both periods: $c_t < d_t, t = 1, 2$
- Katz and Shapiro characterize a number of cases, we focus on one to emphasize the effects of externalities between buyers
- assume that A is an existing technology not protected by a patent (e.g. patent expired or A is open source)
- A is sold at marginal costs in both periods: $p_t = c_t, t = 1, 2$

Katz and Shapiro (1986b) (cont.)

- B is protected by a patent; only one firm can sell B
- this allows B to internalize some externalities (which is not possible for the sellers of A); there exists an equilibrium in which B wins although it is the inferior technology
- Segal (1999): consumers buying B have a negative externality on non-traders (which is here: people that buy A); hence too many people buy B
- Notation: $v_1 = v(N_1), v_2 = v(N_2), v_b = v(N_1 + N_2)$
- Assume

$$d_2 - c_2 < v_b - v_2 \tag{6}$$

$$d_1 - c_1 > 0 \tag{7}$$

$$d_2 - c_2 > 0 \tag{8}$$

Katz and Shapiro (1986b) (cont.)

- if A wins the first period, A wins the second period as well (equation (6) together with (8) imply $v_b - c_2 > v_2 - d_2$)
- choice by first generation has externality on second generation
- Now consider the case where B wins the first period, *check* that charging

$$q_2 = v_b - v_2 + c_2 > d_2 \tag{9}$$

  (inequality follows from (6) makes sure that B wins the second period
- consider the first period:
  - if the first generation chooses A, so will the second generation; utility of first generation: $v_b - c_1$

Katz and Shapiro (1986b) (cont.)

- if B charges $q_1 = c_1 < d_1$, utility of first generation equals $v_b - c_1$ as well; by charging slightly less than c_1, B wins first period for sure

- although B is socially inferior, B sells below costs in the first period to capture N_1
- this makes A a weak competitor in the second period (as $v_b > v_2$) and hence B makes a profit in the second period (to recoup first period losses)
- to see that $q_2 > d_2$, note that equation (6) implies that $v_b - v_2 + c_2 - d_2 > 0$

Kahn and Mookerjee (1998)

- This paper considers the effects if a principal cannot commit to an exclusive contract in an insurance context
- risk averse customer (principal, P) deals with a number of risk neutral insurance companies (agents)
- P cannot commit to stop writing contracts with other insurers
- hence P deals sequentially with insurers
- each insurer understands that its contract with P affects P's incentives to contract with future insurers (i.e. no passive beliefs)

Kahn and Mookerjee (1998) (cont.)

- P can invest effort to avoid the accident state; hence there is a negative externality between insurers: more insurance leads to lower effort and hence higher probability that insurer needs to pay
- exclusive contracts are welfare enhancing in this context

Model

- 2 states of the world $\{1, 2\}$ where state 1 (2) is the no-accident (accident) state
- P can invest effort $e \in \{0, 1\}$ at cost ψe ($\psi > 0$) to increase the probability of the safe state 1 from p_l to $p_h > p_l$
- P invests effort if and only if

$$p_h u(x_1) + (1 - p_h)u(x_2) - \psi \geq p_l u(x_1) + (1 - p_l)u(x_2) \quad (10)$$

where u is increasing and concave

Model (cont.)

- an indifference curve is defined as

$$\max\{p_h u(x_1)+(1-p_h)u(x_2)-\psi, p_l u(x_1)+(1-p_l)u(x_2)\} = const. \tag{11}$$

- Figure 1 shows an indifference curve, denoted uu
- the indifference curve has a point of non-differentiability ("cusp") where P switches from $e=0$ to $e=1$
- the line SS goes through these cusp points
- above this line x_1, x_2 are close enough together such that $e=0$
- below SS we have $e=1$

Model (cont.)

- the line SS denotes the line of second best contracts: highest insurance that P can get, while making sure that P chooses $e = 1$
- FF denotes first best insurance $x_1 = x_2$: if effort were contractible, P could get full insurance
- the red dashed lines are isoprofit lines for the insurer at $e = 0$
- the blue dashed line is an isoprofit line at $e = 1$
- blue line is steeper: same decrease in x_1 leads to bigger increase in x_2 as accident state 2 is less likely to happen

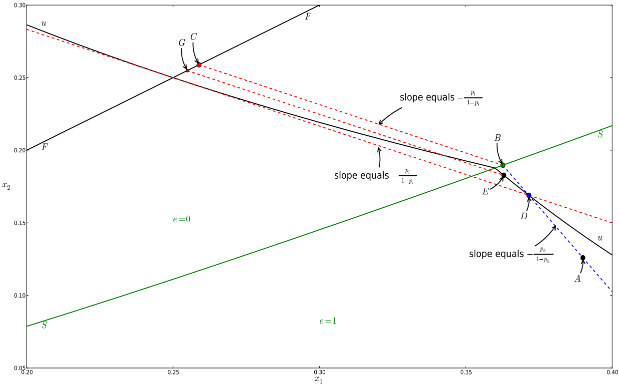


Figure: First, second and third best contracts.

Results

- consider endownment point A
- if P can commit to exclusive contract, he gets the second best contract B
- however, P cannot make such a commitment, the insurer that would offer B, would make a loss:
    - after having signed on for B, P would find another insurer who is willing to offer contract C
    - C offers full insurance and since it lies on the insurer's isoprofit line with slope $p_l/(1-p_l)$ the second insurer does not make a loss (and P strictly benefits)
    - however, this implies that P chooses $e=0$ making contract B loss making for the first insurer
    - contract B is not offered in equilibrium

Results (cont.)

- the first papers in this literature focused on the following equilibrium: full insurance at inefficiently low effort
- to see this, consider endowment point E
  - equilibrium contract is given by G on insurer's isoprofit line with slope $p_l/(1-p_l)$
  - hence G does not lead to a loss
  - no incentive for P to contract with other insurers
  - the inefficiency here is the low effort level
- Kahn and Mookerjee point out that another equilibrium exists
- consider again endownment point A, then equilibrium contract is given by D
  - this leads to nonnegative profits as it lies on isoprofit line with slope $p_h/(1-p_h)$ and $e=1$ (D lies below SS curve)

Results (cont.)

- consumer has no (strictly positive) incentive to contract with another insurer
- indifference curve through D lies everywhere above isoprofit line through D with slope $p_l/(1-p_l)$
- hence no strictly positive surplus can be generated by contracting with further insurers
- D is third best outcome
- although P is indifferent to contract to full insurance, such further contracting would break the equilibrium (as insurer offering D would make a loss)
- hence we use the assumption that P sticks to equilibrium when indifferent

- contract D has efficient effort but leads to underinsurance (even less insurance than second best)

Results (cont.)

- hence inability to commit to exclusive contract leads either to ineffient low effort or to underinsurance
- note that in equilibrium, P will only contract with one insurer
  - hence, if one observes that in reality people only contract with one insurer, this does not mean that exlusive contracts are (costlessly) available
  - further, this does not imply that lack of commitment has no effect on equilibrium outcome
- monopoly insurer can offer a more efficient second best contract
- however, monopolist will also appropriate more of the surplus
- not clear that monopoly insurer is better for P than competitive insurance market

Part III

Offer game and beliefs

Beliefs in offer games

- HT90 assume passive beliefs: upstream monopolist cannot commit to supplying the monopoly output level (in total)
- U has an incentive to deviate to higher output level than $q^m/2$ with downstream firm
- equilibrium output level equals Cournot outcome
- *Check* that with symmetric beliefs upstream monopolist can implement the monopoly output (without using exclusive contracts) [hint: check what happens if upstream firm deviates and offers one downstream firm a higher output level (than $q^m/2$)]

Robust predictions

- HT90 outcome depends on the assumption of (passive) beliefs
- Can we come up with predictions independent of beliefs?
- SW03: identify properties of equilibrium outcomes that are robust in the sense that they must be satisfied by all equilibria of all bilateral contracting games
- idea is to consider contracting games where parties can offer each other a menu-contract from which the principal can then choose (rather than a "point contract")
- Introducing such a menu contract bounds the set of possible outcomes
- Only outcomes between the pairwise stable outcome (Cournot) and the competitive outcome (price equals marginal costs) can be an equilibrium

Model

- Retailer i has profit: $U_i = P(X)x_i - t_i$ where $P(.)$ denotes inverse demand as a function of $X = \sum_{i \in N} x_i$ and t_i denotes transfer from i to manufacturer M
- In other words, i sells without (further) costs
- M's profit: $U_M = \sum_{i \in N} t_i - c(X)$
- Assume $c(0) = 0, c'(.), c''(.) > 0$
- define competitive outcome p^c, X^c as
  - $\{X^c\} = \arg\max_X p^c X - c(X)$
  - $p^c = P(X^c)$
- we assume that competitive outcome exists and is unique
- further assume $p^c X - c(X)$ is strictly decreasing for $X \geq X^c$ and $P(X) \leq p^c$ for each $X \geq X^c$

Model (cont.)

- Contracting game: lasts for K periods. In each period k, a subset $A_k \subset N$ of retailers simultaneously offers menus to M and simultaneously M offers menus to $M_k \subset N$ retailers
- M and retailers simultaneously decide whether to accept the contracts offered to them
- Assume $\cup_{k=1}^{K}(A_k \cup M_k) = N$
- M observes entire history, retailer i only observes the offers made to him and whether M accepted/rejected his offers to M
- At the end of the game M chooses (x_i, t_i) from the last contract accepted with retailer i and $(0, 0)$ if no contract has been accepted with i
- We consider pure-strategy (weak) Perfect Bayesian Equilibrium (PBE): FT page 325, Mas-Colell et al. page 285

Model (cont.)

- parties can offer singleton contracts or menus; for reasons that become clear below we focus on the competitive menu:

$$C = \{(x, p^c x) | x \in [0, X^c]\} \tag{12}$$

- (IR) constraint for retailer i: $P(X)x_i - t_i \geq 0$
- Define the function $\Pi^c(X_{-i})$ as

$$\Pi^c(X_{-i}) = \begin{cases} p^c(X^c - X_{-i}) - c(X^c) & \text{if } X_{-i} \leq X^c \\ -c(X_{-i}) & \text{otherwise} \end{cases} \tag{13}$$

- Which outcomes can be sustained in equilibrium?

Menu deviation condition

Proposition 3

In any PBE $(\hat{x}, \hat{t})$ it must be the case that for each retailer $i \in N$ the following condition holds:

$$P(\hat{X})\hat{x}_i - c(\hat{X}) \geq \Pi^c(\hat{X}_{-i}) \tag{MD}$$

- Interpretation: the equilibrium joint profit of M and i is higher than the profit they can get if M offers i the competitive menu C and then chooses its profit maximizing point from C (*check* that RHS corresponds to this choice from C)
- **Proof**: Suppose not; i.e. there exists $i \in N$ such that

$$P(\hat{X})\hat{x}_i - c(\hat{X}) < \Pi^c(\hat{X}_{-i}) \tag{14}$$

Menu deviation condition (cont.)

- let $\bar{k}$ denote the last period in which M and i have a contracting opportunity
- Suppose that M makes the offer in this period:
  - M deviates by only offering $(0, 0)$ to i prior to $\bar{k}$ and by rejecting all offers from i
  - in period $\bar{k}$, M offers i menu C with t_i that is $\varepsilon > 0$ below the corresponding competitive transfer: i makes a positive profit (ε) with this menu and accepts (Note: this is the point where normally we have to invoke i's beliefs; why are beliefs not relevant here?)

Menu deviation condition (cont.)

- M sticks to equilibrium strategies with other players (perhaps M can do even better) and gets payoff:

$$\sum_{j \neq i} \hat{t}_j + \Pi^c(\hat{X}_{-i}) - \varepsilon > \sum_{j \neq i} \hat{t}_j + P(\hat{X})\hat{x}_i - c(\hat{X}) \qquad (15)$$

$$\geq \sum_{j \in N} \hat{t}_j - c(\hat{X}) \qquad (16)$$

- second inequality follows because in hypothesized equilibrium i will not pay more than $P(\hat{X})\hat{x}_i$

- Suppose that i makes offer in $\bar{k}$
  - i offers M the menu C minus payment

$$\Delta = \Pi^c(\hat{X}_{-i}) - (\hat{t}_i - c(\hat{X})) - \varepsilon$$

that is, $t_i = p^c(X^c - \hat{X}_{-i}) - \Delta$

Menu deviation condition (cont.)

- if accepted, the retailer gets

$$p^c(X^c - \hat{X}_{-i}) - t_i = \Delta > P(\hat{X})\hat{x}_i - \hat{t}_i$$

- *check* that the last inequality follows (for $\varepsilon > 0$ small enough) from the definition of Δ and equation (14)
- The principal gains as well because:

$$t_i - c(X^c) > \hat{t}_i - c(\hat{X})$$

- *check* that this is true for $\varepsilon > 0$ $\square$

Pairwise stability

- We say that $\hat{x}$ is pairwise stable if

$$\hat{x}_i \in \arg\max_x P(x + \hat{X}_{-i})x - c(x + \hat{X}_{-i}) \qquad (17)$$

  for each $i \in N$
- Note that $\hat{X}_{-i}$ is taken as given here: it is as if M and i consider their joint surplus where i has passive beliefs
- Note that pairwise stability is stronger than (MD) because (MD) only checks for deviations using menu C

Proposition 4

Any pairwise stable $\hat{x}$ satisfies (MD)

Pairwise stability (cont.)

- **Proof** Suppose not; i.e. $\hat{x}$ is pairwise stable but there exists i such that
$$P(\hat{X})\hat{x}_i - c(\hat{X}) < \Pi^c(\hat{X}_{-i})$$
then we have that
$$p^c(X^c - \hat{X}_{-i}) - c(X^c) > P(\hat{X})\hat{x}_i - c(\hat{X})$$
or equivalently
$$P(x + \hat{X}_{-i})x - c(x + \hat{X}_{-i}) > P(\hat{X})\hat{x}_i - c(\hat{X})$$
for $x = X^c - \hat{X}_{-i}$.
- This contradicts that $\hat{x}$ is pairwise stable. $\square$

Convergence to competitive outcome

Proposition 5

If $\{\hat{X}^N\}_{N=1}^{\infty}$ is a sequence of PBE aggregate trades in a sequence of bilateral contracting games with N retailers, then:

(a) $\hat{X}^N \leq X^c$ *for all* N

(b) *if* $W(X) \equiv P(X)X - c(X)$ *is bounded above on* $X \in \mathbb{R}_+$, *then* $\hat{X}^N \to X^c$ *as* $N \to \infty$

Convergence to competitive outcome (cont.)

- **Proof.** Suppose –by contradiction– that (a) does not hold; i.e. $\hat{X}^N > X^c$ for some N. Because $P(X)$ is falling in X for $X > X^c$ we have

$$P(\hat{X}^N)\hat{x}_i^N - c(\hat{X}^N) < p^c \hat{x}_i^N - c(\hat{X}^N) \tag{18}$$

$$\leq \max_x p^c x - c(\hat{X}_{-i}^N + x) \tag{19}$$

$$= \begin{cases} -c(\hat{X}_{-i}^N) & \text{if } \hat{X}_{-i}^N > X^c \\ p^c(X^c - \hat{X}_{-i}^N) - c(X^c) & \text{otherwise} \end{cases} \tag{20}$$

  which contradicts proposition 3.

- Part (a) implies that $\hat{X}_{-i}^N \leq \hat{X}^N \leq X^c$
- hence we have $\Pi^c(\hat{X}_{-i}^N) = p^c(X^c - \hat{X}_{-i}^N) - c(X^c)$.

Convergence to competitive outcome (cont.)

- Adding up (MD) over $i \in N$ we get

$$P(\hat{X}^N)\hat{X}^N - Nc(\hat{X}^N) \geq p^c(NX^c - (N-1)\hat{X}^N) - Nc(X^c)$$

or equivalently (*check*)

$$p^c\hat{X}^N - c(\hat{X}^N) \geq \frac{N}{N-1}(p^cX^c - c(X^c)) - \frac{1}{N-1}W(\hat{X}^N) \tag{21}$$

$$\geq \frac{N}{N-1}(p^cX^c - c(X^c)) - \frac{1}{N-1}\sup_X W(X) \tag{22}$$

$$\to p^cX^c - c(X^c) \text{ as } N \to \infty \tag{23}$$

- Hence $\hat{X}^N \to X^c$ as $N \to \infty$ because $X^c = argmax_X p^cX - c(X)$ $\square$

What is so special about menu C?

Lemma 2

Consider the set of menus $(x_i(X_{-i}), P(x_i(X_{-i}) + X_{-i})x_i(X_{-i}))$ where $x_i'(X_{-i})$ exists. Out of this set of menus, the profit maximizing menu for M is the competitive menu C.

- **Proof.** M chooses the profit maximizing point out of the menu:

$$\Pi_i(X_{-i}) = \max_{t_i, x_i}\{t_i - c(x_i + X_{-i})\}$$

envelope theorem implies (see (IC) in equation (1))

$$\Pi_i'(X_{-i}) = -c'(x_i + X_{-i}) \tag{24}$$

What is so special about menu C? (cont.)

- We also have

$$\Pi_i(X_{-i}) = x_i(X_{-i})P(x_i(X_{-i}) + X_{-i}) - c(x_i(X_{-i}) + X_{-i})$$

- taking the derivative and setting it equal to equation (24) yields

$$-c'(x_i(X_{-i}) + X_{-i}) = x_i'(X_{-i})P(x_i(X_{-i}) + X_{-i}) + (x_i'(X_{-i}) + 1)(x_i(X_{-i})P'(x_i(X_{-i}) + X_{-i}) - c'(x_i(X_{-i}) + X_{-i})) \tag{25}$$

- which is a differential equation for x_i with boundary condition $x_i(X^c) = 0$
- *check:* $x_i(X_{-i}) = X^c - X_{-i}$ solves this differential equation
- this solution for x_i is implemented by menu C. $\square$

Characterization of PBE trades $\hat{x}$

Proposition 6

$(\hat{x}, \hat{t})$ is a PBE outcome if and only if it satisfies (IR) and (MD).

- **Proof.** Necessity of (MD) follows from proposition 3. If (IR) is not satisfied, retailers will reject the outcome.
- For sufficiency we need to verify that M cannot benefit from a multilateral deviation to a group D of retailers
- Since for each retailer the optimal deviation uses C, the max. profit M can get from such a deviation equals

$$\begin{aligned}
\Pi_D^C(\hat{X}_{-D}) &= \max_{(x_D, t_D)\in C^{|D|}} \sum_{i\in D} t_i - c(X_D + \hat{X}_{-D}) \\
&= p^c(X^c - \hat{X}_{-D}) - c(X^c) \\
&= \pi^c - p^c(\hat{X} - \hat{X}_D)
\end{aligned}$$

Characterization of PBE trades $\hat{x}$ (cont.)

- summing (MD) over $i \in D$ and noting that $\sum_{i \in D} X_{-i} = \sum_{i \in D}(X - X_i) = |D|X - X_D$ we get

$$
\begin{aligned}
P(\hat{X}) \sum_{i \in D} \hat{x}_i - |D|c(\hat{X}) &\geq |D|\pi^c - |D|p^c\hat{X} + p^c\hat{X}_D \\
&= \pi^c - p^c(\hat{X} - \hat{X}_D) + (|D| - 1)(\pi^c - p^c\hat{X}) \\
&\geq \pi^c - p^c(\hat{X} - \hat{X}_D) - (|D| - 1)c(\hat{X})
\end{aligned}
$$

  because $\pi^c \geq p^c\hat{X} - c(\hat{X})$.
- Hence we find that

$$
P(\hat{X})\hat{X}_D - c(\hat{X}) \geq \pi^c - p^c(\hat{X} - \hat{X}_D) = \Pi_D^C(\hat{X}_{-D})
$$

- In words, (MD) implies that multilateral deviation (to contract C) is not profitable for M

Characterization of PBE trades $\hat{x}$ (cont.)

- A PBE sustaining $(\hat{x}, \hat{t})$ takes the following form:
  - M offers $(\hat{x}_i, \hat{t}_i)$ to each retailer i
  - retailer i offers $(\hat{x}_i, \hat{t}_i)$ to M
  - retailer i accepts $(\hat{x}_i, \hat{t}_i)$ and any other menu that gives i non-negative profits for sure (best such menu from M's point of view is the C menu)
  - Any other offer $(\tilde{x}_i, \tilde{t}_i)$ to i leads to beliefs $\tilde{X}_{-i}$ such that

$$P(\tilde{x}_i + \tilde{X}_{-i})\tilde{x}_i - \tilde{t}_i < 0$$

  - and is rejected. $\square$

Conclusions

- HT90 find that with private offers and passive beliefs the Cournot outcome is the equilibrium; monopoly profits cannot be sustained in equilibrium
- However, passive beliefs are not always convincing and other (e.g. symmetric) beliefs lead to other outcomes
- SW03 allow parties to offer each other a menu of contracts from which M can choose
- This narrows down the possible equilibrium outcomes: only outcomes between the bilaterally stable one (Cournot) and the competitive outcome are robust to the introduction of such menus of contracts

Part IV

Bidding games and menu auctions

Menu auction model

- Auctioneer M and set of bidders $\Im$
- J denotes subset of bidders and $\bar{J}$ is its complement
- possible allocations $s \in S$ where S is a finite set
- M's cost of implementing s equals $d(s)$
- j's utility from s is $g_j(s)$
- capital G denotes a sum: $G_J(s) = \sum_{j \in J} g_j(s)$
- set of allocations that maximize utility of J bidders and M

$$S^J = \arg\max_{s \in S} G_J(s) - d(s) \tag{26}$$

- $S^* = S^{\Im}$ set of efficient allocations for M and $\Im$ together
- each bidder j makes contingent bids $f_j(s) \geq 0$

Menu auction model (cont.)

- M implements allocation s that maximizes her utility $u_M(s) = \sum_{j \in \Im} f_j(s) - d(s)$.
- Set of such allocations:

$$I^*(\{f_j\}_{j \in \Im}) = \arg\max_{s \in S} u_M(s) \tag{27}$$

- Utility bidder j is denoted $u_j(s) = g_j(s) - f_j(s)$
- With sums: $U_J(s) = \sum_{j \in J} u_j(s) = G_J(s) - F_J(s)$
- $(\{f_j\}_{j \in \Im}, s^0)$ is a *Nash equilibrium* if
  - $f_j(s^0) \geq 0$ for all $j \in \Im$ and $s \in S$
  - $s^0 \in I^*(\{f_j\}_{j \in \Im})$
  - No bidder j has a strategy $\tilde{f}_j \geq 0$ that would yield higher utility than $u_j(s^0)$

Nash equilibrium

Lemma 3

$(\{f_j\}_{j\in\Im}, s^0)$ is a Nash equilibrium if and only if

1. $f_j \geq 0$ *for all* $j \in \Im$
2. $s^0 \in I^*(\{f_j\}_{j\in\Im})$*:* s^0 *maximizes* M*'s utility*
3. $g_j(s_0) + F_{-j}(s^0) - d(s^0) \geq g_j(s) + F_{-j}(s) - d(s)$ *for all* $j \in \Im$ *and* $s \in S$*: coalition of* i *and* M *cannot gain by deviating to* $s \neq s^0$
4. *for every* $j \in \Im$ *there exists* $s^j \in I^*(\{f_j\}_{j\in\Im})$ *such that* $f_j(s^j) = 0$

- **Proof.** *Necessity*
  1. by assumption bids must be non-negative
  2. M maximizes payoffs in equilibrium

Nash equilibrium (cont.)

3. Suppose not, i.e. there exists j and $\tilde{s}$ such that

$$g_j(\tilde{s}) - \left([F_{\Im}(s^0) - d(s^0)] - [F_{-j}(\tilde{s}) - d(\tilde{s})]\right) > g_j(s^0) - f_j(s^0) \tag{28}$$

then j can deviate to

$$\tilde{f}_j(\tilde{s}) = [F_{\Im}(s^0) - d(s^0)] - [F_{-j}(\tilde{s}) - d(\tilde{s})] + \varepsilon$$

for $\varepsilon > 0$ sufficiently small. *Check* that M will choose $\tilde{s}$ because

$$U_M = \tilde{f}_j(\tilde{s}) + F_{-j}(\tilde{s}) - d(\tilde{s}) > F_{\Im}(s^0) - d(s^0)$$

and *check* that j gains because

$$U_j = g_j(\tilde{s}) - \tilde{f}_j(\tilde{s}) > g_j(s^0) - f_j(s^0)$$

Nash equilibrium (cont.)

4. if such s^j does not exist for j, j can lower $f_j(s)$ for each $s \in I^*(\{f_j\}_{j\in\Im})$. This will not change M's choice of s but clearly j's payoff increases.

- *Sufficiency* Suppose that $(\{f_j\}_{j\in\Im}, s^0)$ satisfies conditions 1-4 but is not a Nash equilibrium. Then there exists bidder j and strategy $\tilde{f}_j$ such that this deviating strategy induces M to choose $\tilde{s}$:

$$\tilde{f}_j(\tilde{s})+F_{-j}(\tilde{s})-d(\tilde{s}) \geq \tilde{f}_j(s)+F_{-j}(s)-d(s) \text{ for all } s \in S \quad (29)$$

and j strictly gains:

$$g_j(\tilde{s}) - \tilde{f}_j(\tilde{s}) > g_j(s^0) - f_j(s^0) \quad (30)$$

Nash equilibrium (cont.)

Combining this equation with condition 3 yields

$$\tilde{f}_j(\tilde{s}) - f_j(s^0) < g_j(\tilde{s}) - g_j(s^0) \leq F_{-j}(s^0) - F_{-j}(\tilde{s}) - d(s^0) + d(\tilde{s})$$

Or equivalently

$$\tilde{f}_j(\tilde{s}) < [F_{\Im}(s^0) - d(s^0)] - [F_{-j}(\tilde{s}) - d(\tilde{s})]$$

by condition 4 there exists s^j such that

$$0 + F_{-j}(s^j) - d(s^j) = F_{\Im}(s^0) - d(s^0)$$

hence we have

$$\tilde{f}_j(\tilde{s}) < [\tilde{f}_j(s^j) + F_{-j}(s^j) - d(s^j)] - [F_{-j}(\tilde{s}) - d(\tilde{s})]$$

Nash equilibrium (cont.)

because $\tilde{f}_j \geq 0$. Rewriting we have

$$\tilde{f}_j(\tilde{s}) + F_{-j}(\tilde{s}) - d(\tilde{s}) < \tilde{f}_j(s^j) + F_{-j}(s^j) - d(s^j)$$

which contradicts equation (29) for $s = s^j$. $\square$

Truthful strategies

- f_j is a *truthful strategy* relative to s^0 if either
  - $g_j(s) - f_j(s) = g_j(s^0) - f_j(s^0)$ or
  - $g_j(s) - f_j(s) \leq g_j(s^0) - f_j(s^0)$ and $f_j(s) = 0$
- $(\{f_j\}_{j\in\Im}, s^0)$ is a *truthful Nash equilibrium* if it is a Nash equilibrium and $\{f_j\}_{j\in\Im}$ are truthful strategies relative to s^0.
- In a truthful equilibrium, each bidder reveals his net willingness to pay for s relative to s^0
- The following result shows that bidders (and we) can restrict attention to truthful strategies

Theorem 4

For any set of offers $\{f_j\}_{j\neq i}$, i's best response contains a truthful strategy

Truthful strategies (cont.)

- Suppose one of i's optimal responses is f_i and with this response M chooses s^0. Further, assume that f_i is not truthful relative to s^0. Define strategy $\tilde{f}_i$ such that $\tilde{f}_i(s^0) = f_i(s^0)$ and $\tilde{f}_i$ is truthful relative to s^0. Two possibilities:
  - if M chooses s^0 under $\tilde{f}_i$ as well, i does not loose from switching to $\tilde{f}_i$
  - if M switches to $\tilde{s} \neq s^0$, it must be the case that $\tilde{f}_i(\tilde{s}) > f_i(\tilde{s}) \geq 0$. But then i cannot loose from using the truthful strategy $\tilde{f}_i$ because

$$g_i(\tilde{s}) - \tilde{f}_i(\tilde{s}) = g_i(s^0) - \tilde{f}_i(s^0) = g_i(s^0) - f_i(s^0)$$

$\square$

Utilities for bidders

Theorem 5

In each truthful Nash equilibrium, M chooses $s^0 \in S^*$. For each coalition $J \subset \Im$, bidders payoffs satisfy

$$\sum_{j\in J}\{g_j(s^0) - f_j(s^0)\} \leq \sum_{j\in\Im} g_j(s^0) - d(s^0) - \max_{s\in S}\{\sum_{j\notin J} g_j(s) - d(s)\} \tag{31}$$

- Equation (31) says that a coalition J cannot get more than its marginal contribution to the "grand coalition"
- We can also write this as saying that the coalition of M and $\Im \setminus J$ cannot gain by deviating (excluding J):

$$\sum_{j\notin J} g_i(s^0) + \sum_{j\in J} f_j(s^0) - d(s^0) \geq \max_{s\in S}\{\sum_{j\notin J} g_j(s) - d(s)\}$$

Utilities for bidders (cont.)

- The intuition why $s^0 \in S^*$ is that in a truthful equilibrium M takes the marginal valuations of all bidders into account
- **Proof.** First, suppose that M's equilibrium choice $s^0 \notin S^*$. Then because strategies are truthful relative to s^0, we have for $s^* \in S^*$ that

$$f_j(s^*) \geq g_j(s^*) - [g_j(s^0) - f_j(s^0)]$$

- summing over all j:

$$F_{\mathfrak{J}}(s^*) \geq G_{\mathfrak{J}}(s^*) - G_{\mathfrak{J}}(s^0) + F_{\mathfrak{J}}(s^0)$$

Utilities for bidders (cont.)

- Therefore

$$F_{\Im}(s^*) - d(s^*) \geq \underbrace{[G_{\Im}(s^*) - d(s^*)] - [G_{\Im}(s^0) - d(s^0)]}_{>0 \text{ because } s^* \in S^* \text{ and } s^0 \notin S^*} + [F_{\Im}(s^0) - d(s^0)] > F_{\Im}(s^0) - d(s^0) \quad (32)$$

- which contradicts that M chooses s^0 in equilibrium
- Now consider equation (31). Using equation (26) and the fact that M chooses s^0 in equilibrium, we have that

$$F_{\Im}(s^0) - d(s^0) \geq F_{\Im}(s^{\bar{J}}) - d(s^{\bar{J}})$$

Utilities for bidders (cont.)

- Therefore

$$
\begin{aligned}
F_J(s^0) + F_{\bar{J}}(s^0) - d(s^0) &\geq F_J(s^{\bar{J}}) + F_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}}) && (33)\\
&\geq F_{\bar{J}}(s^{\bar{J}}) - d(s^{\bar{J}}) && (34)\\
&\geq [G_{\bar{J}}(s^{\bar{J}}) - G_{\bar{J}}(s^0) + F_{\bar{J}}(s^0)] - d(s^{\bar{J}}) && (35)
\end{aligned}
$$

- where the second line follows from $F_J(s^{\bar{J}}) \geq 0$ and the third from the fact that f_j is truthful relative to s^0.
- *check* that this implies (31). $\square$

Menu auction in HT90

- Consider the Hart-Tirole game with two retailers and one manufacturer M
- Consider a menu auction in which retailers make offers to manufacturer
- note that these offers depend on the final allocation: on x_i and x_{-i}.
- Assume demand takes the form $p_i(x_i, x_{-i})$, retailers sell without further costs
- M produces output at costs $c(x_1 + x_2)$
- Truthful bids imply that they take the form

$$t_i(x_i, x_{-i}) = p(x_i, x_{-i})x_i - \bar{u}_i$$

for some $\bar{u}_i \geq 0$

Menu auction in HT90 (cont.)

- M solves

$$\max_{x_1, x_2} [p(x_1, x_2)x_1 - \bar{u}_1] + [p(x_2, x_1)x_2 - \bar{u}_2] - c(x_1 + x_2)$$

- Hence the efficient (monopoly) output levels are chosen
- Since the coalition M and R_2 can deviate and exclude R_1 we have

$$[p(x_1, x_2)x_1 - \bar{u}_1] + p(x_2, x_1)x_2 - c(x_1 + x_2) \geq \max_{x_2} p(x_2, 0)x_2 - c(x_2)$$

- or equivalently

$$\bar{u}_1 \leq [p(x_1, x_2)x_1 + p(x_2, x_1)x_2 - c(x_1 + x_2)] - \max_{x_2} p(x_2, 0)x_2 - c(x_2)$$

- R_1 does not get more than his marginal contribution to the surplus

Common agency

- BW86 assume that retailer i's offer can depend on x_{-i}
- Now we assume that i's offer can only depend on x_i
- Two retailers (as principals) make offers to common agent M: bidding game
- two types of common agency games:
  - *intrinsic*: M either accepts both offers or rejects both offers from retailers 1 and 2
  - *delegated*: M can either accept no, one or both offers
- two types of contract:
  - singleton contracts with just the equilibrium offer
  - menu contracts with equilibrium offer and out-of-equilibrium offers

Common agency (cont.)

	intrinsic	delegated
singleton	$q = q^c$, $U_M = 0$	one retailer, $q = 2q^m$, $U_M > 0, U_1 = U_2 = 0$
menu	$q \in [q^c, q^b]$, $U_M = 0$	propos. 5 in MS03

Table: Summary of results

- Note that in MS03 q^c denotes Cournot output and q^b denotes competitive output level
- With intrinsic common agency:
  - singleton contracts lead to Cournot outcome

Common agency (cont.)

- with extended menu contracts, there are multiple equilibria with q^c as lower bound on output
- hence it is a bit strange that retailers would use extended contracts at all => MS03 section 5 presents a model with asymmetric information about M's efficiency. Then a contract is offered for each type of M and extended contracts make sense

- With delegated common agency and singleton contracts:
    - there is no equilibrium in which both retailers serve M. Under some condition there is an equilibrium in which retailers compete rents away to become exclusive retailer of M, selling the monopoly output.
- In these lectures we will not cover the case with delegated common agency and extended menu of contracts: see proposition 5 in the paper

Model

- two retailers sell homogeneous good bought from M at transfer t_i and no further costs
- inverse demand: $P(Q)$ where $Q = q_1 + q_2$ with $P(0) > 0, |P'(0)| < +\infty, P' < 0, P'' \leq 0$
- M produces at cost $c(Q)$ (we focus on $\theta = 1$) with $c(0) = c'(0) = 0, c', c'', c''' > 0$
- Retailer i's profit: $U_i = P(q_i + q_{-i})q_i - t_i$
- M's profit: $U_M = t_1 + t_2 - C(q_1 + q_2)$; M's outside option equals 0 (zero)
- M and i can only contract on q_i not on q_{-i}
- we consider pure strategy symmetric differentiable (in case of menu) equilibria

Model (cont.)

- Benchmark *per firm* symmetric output levels:
  - competitive: $P(2q^b) = c'(2q^b)$
  - Cournot: $P(2q^c) + q^c P'(2q^c) = c'(2q^c)$
  - monopoly: $P(2q^m) + 2q^m P'(2q^m) = c'(2q^m)$
  - *check* that: $0 < q^m < q^c < q^b$
  - we assume that aggregate profits $2q^i P(2q^i) - c(2q^i) \geq 0$ for $i = m, c, b$

Intrinsic C.A. with singleton contracts

- contracts are of the form (q_i, t_i)

Proposition 7

Cournot output q^c is the unique (pure strategy symmetric) equilibrium

Proof.

- consider R_1's problem: M's (IR) constraint is

$$t_1 + t_2 - c(q_1 + q_2) \geq 0 \tag{36}$$

- no reason to leave rents to M, hence R_1's optimization problem is

$$\max_{q_1} P(q_1 + q_2)q_1 - (c(q_1 + q_2) - t_2)$$

Intrinsic C.A. with singleton contracts (cont.)

- first order condition:

$$P'(q_1 + q_2)q_1 + P(q_1 + q_2) - c'(q_1 + q_2) = 0$$

- by assumptions above second derivative w.r.t. q_1 is negative
- symmetric solution is $q_1 = q_2 = q^c$
- this is the same outcome as in the HT90 offer game, but now we do not need (passive) beliefs: R_1 –as part of the Nash equilibrium– takes (q_2, t_2) as given $\square$

Intrinsic C.A. with menus

- R_i offers M a menu $(q_i, T(q_i))$ and M chooses from this menu
- in equilibrium M chooses only one combination (q_i, t_i) so why are the other (out-of-equilibrium) choices relevant?
- Suppose R_1 would like to deviate from equilibrium by changing q_1, which q_2 would M choose?

$$q_2^*(q_1) = \arg\max_{q_2} T_2(q_2) - c(q_1 + q_2) \tag{37}$$

- if "things are concave", q_2^* solves

$$T_2'(q_2^*(q_1)) = c'(q_1 + q_2^*(q_1)) \tag{38}$$

- Hence out of equilibrium choices affect the incentives for the other retailer to deviate

Intrinsic C.A. with menus (cont.)

- Intuitively, if $\partial q_2^*(q_1)/\partial q_1 < 0$ is large in absolute value, big incentive for R_1 to raise q_1
- Hence lowest equilibrium output if q_2 does not fall at all with q_1: outcome with singleton contracts
- put differently, when q_2 hardly increases if q_1 falls, R_1 has big incentive to reduce q_1, leads to (relatively) low equilibrium output

Proposition 8

1. *only $q \in [q^c, q^b]$ can be an equilibrium outcome*
2. *each $q \in [q^c, q^b]$ can be an equilibrium outcome*

In each equilibrium, M gets zero profits: $U_M = 0$

Intrinsic C.A. with menus (cont.)

1. **Proof.** Take $T_2(q_2)$ as given and assume that $T_2(q_2) - c(q_1 + q_2)$ is concave in q_2
   - R_1 has to take (IR) into account:

$$t_1 + T_2(q_2^*(q_1)) - c(q_1 + q_2^*(q_1)) \geq 0$$

   - no reason to leave rents to M, hence R_1's optimization problem is to maximize

$$V(q_1) = P(q_1 + q_2^*(q_1))q_1 + T_2(q_2^*(q_1)) - c(q_1 + q_2^*(q_1))$$

   - Using envelope argument, first order condition q_1 can be written as

$$q_1 P'(q_1+q_2^*(q_1))\left(1+\frac{\partial q_2^*}{\partial q_1}\right)+P(q_1+q_2^*(q_1))-c'(q_1+q_2^*(q_1)) = 0 \tag{39}$$

Intrinsic C.A. with menus (cont.)

- Use equation (38) to solve for

$$\frac{\partial q_2^*}{\partial q_1} = \frac{c''}{T_2'' - c''} \leq 0 \qquad (40)$$

- substitute this into (39) and imposing symmetric solution $q_1 = q_2 = q$ and $T_1(q) = T_2(q) = T(q)$ yields

$$P(2q) + qP'(2q) = c'(2q) - \frac{qP'(2q)c''(2q)}{T''(q) - c''(2q)} \qquad (41)$$

- M solves

$$\max_{q_1, q_2} T(q_1) + T(q_2) - c(q_1 + q_2) \qquad (42)$$

Intrinsic C.A. with menus (cont.)

- For this problem to be concave, we need the Hessian to be negative semi-definite:

$$T''(q) - c''(2q) \leq 0 \tag{43}$$

$$(T''(q) - c''(2q))^2 \geq (c''(2q))^2 \tag{44}$$

- Substitute (43) into (41):

$$P(2q) + qP'(2q) \leq c'(2q)$$

- Hence $q \geq q^c$
- Now substitute (44) into (41):

$$P(2q) \geq c'(2q)$$

- Hence $q \leq q^b$
- Therefore only $q \in [q^c, q^b]$ can be part of an equilibrium

Intrinsic C.A. with menus (cont.)

2. To show that each $q \in [q^c, q^b]$ can be part of an equilibrium, we only need to give an example of a tariff function $T(q)$ that gives such a result
   - Consider $T(q) = \alpha + \beta q + \frac{1}{2}\gamma q^2$
   - Then M's first order condition can be written as

$$\beta + \gamma q = c'(2q)$$

   - R_i's first order condition (41) needs to be satisfied:

$$P(2q) + qP'(2q) = c'(2q) - \frac{qP'(2q)c''(2q)}{\gamma - c''(2q)} \tag{45}$$

   - M gets zero rents:

$$2(\alpha + \beta q + \tfrac{1}{2}\gamma q^2) = c(2q)$$

Intrinsic C.A. with menus (cont.)

- these are three equations in 3 unknowns $(\alpha.\beta,\gamma)$ that can be solved (see paper for the solution)
- M's problem is concave if (43) and (44) hold: is true if and only if $\gamma \leq 0$
- ranging from $\gamma = 0$ to $\gamma = -\infty$, equation (45) yields q ranging from q^b to q^c
- *check* for yourself that R_i's optimization problem is concave, i.e. $\partial^2 V(q_i)/\partial q_i^2 < 0$ where (from (40)) $\partial q_2^*/\partial q_1 = c''/(\gamma - c'')$ □

Delegated C.A. with singleton contracts

- Since M can decide to accept only one contract, M earns a positive surplus in equilibrium
- In fact, M earns the whole surplus!
- retailers cannot offer "explicit" exclusive contracts; retailers just offer contracts and M decides which to accept
- this leads retailers to compete fiercely to get their contract accepted by M: they compete their surplus away
- to make their offer as attractive as possible, they offer a contract with the (total) monopoly output
- pure strategy symmetric equilibrium only exists if M does not want to accept both contracts: condition (46) below

Delegated C.A. with singleton contracts (cont.)

Proposition 9

1. *there is no symmetric pure strategy equilibrium in which M sells to both retailers*
2. *If*

$$4q^m P(2q^m) - c(4q^m) < 2q^m P(2q^m) - c(2q^m) \tag{46}$$

there exists a pure strategy equilibrium in which M sells to R_i with probability $\frac{1}{2}$, M produces $2q^m$ receives transfer $t_i = 2q^m P(q^m)$. Retailers make zero profits and $U_M = 2q^m P(2q^m) - c(2q^m) > 0$

1. **Proof.** Suppose that such a symmetric pure strategy equilibrium does exist: (q, t):

Delegated C.A. with singleton contracts (cont.)

- in this (hypothesized) equilibrium, retailers choose t as low as possible to satisfy

$$\begin{aligned} U_M &= 2t - c(2q) \geq 0 \\ U_M &\geq t - c(q) \end{aligned}$$

- that is, M should not want to
  - either reject both contracts
  - or reject one contract
- the second inequality is binding. Suppose not, i.e. assume $2t - c(2q) = 0$. Then $t = c(2q)/2 > c(q)$ by convexity of c. But this implies $U_M = 0 < t - c(q)$ violating the second inequality
- With the second inequality binding we find

$$t = c(2q) - c(q) \tag{47}$$

Delegated C.A. with singleton contracts (cont.)

- Now suppose R_1 offers $q_1 > q$ and $t_1 = c(q_1 + q) - c(q)$
- then M sells to R_1 only because:

$$t_1 - c(q_1) > t_1 + t - c(q_1 + q)$$

- *check* that this follows from equation (47), $q_1 > q$ and convexity of c.
- *check* that accepting R_1's contract only is better than accepting R_2's contract only
- because M sells to R_1 only, this deviation to (q_1, t_1) is profitable since

$$q_1 P(q_1) - (c(q_1 + q) - c(q)) > qP(2q) - (c(2q) - c(q))$$

- $q_1 = q + \varepsilon$ for $\varepsilon > 0$ small increases the transfer in a continuous way, but revenue jumps up for $q > 0$.

Delegated C.A. with singleton contracts (cont.)

2. since M serves at most one retailer, retailers compete to become this retailer
   - to generate highest surplus, each retailer offers $(2q^m, t)$
   - with Bertrand competition in t between retailers, each retailer offers $t = 2q^m P(2q^m)$
   - M is indifferent between retailers and randomizes
   - condition (46) makes sure that M does not accept both contracts
   - since monopoly profit is assumed to be positive, (46) can only hold if c is "pretty convex" $\square$

Conclusion

- In this lecture we considered two bidding games
- hence beliefs play no role in this lecture
- in BW86: contract between M and R_i can depend on (x_i, x_{-i})
- if retailers use truthful strategies, M internalizes retailers' marginal valuations between different allocations and an efficient outcome is chosen in equilibrium
- parties cannot get a payoff higher than their marginal contribution to total surplus
- MS03: R_i's contract cannot depend on x_{-i}
- if a retailer can use a menu of contracts, all outcomes between pairwise stable and competitive outcome can be sustained as equilibrium (same as SW03)

Conclusion (cont.)

- if only singleton contracts can be used, Cournot outcome is equilibrium
- with delegated common agency:
    - equilibrium implies exclusive dealing (without explicit exclusive clauses in the contracts)
    - M gets strictly positive rent

Part V

Excusive dealing

Bernheim and Whinston (1998): Motivation

- Chicago school: the use of exclusionary contracts to foreclose entry and hence reduce competition and welfare cannot happen: such a contract is not accepted in equilibrium
- AB87: M and R can use such a contract to extract rents from entrant
- RRW91: such a contract is accepted in equilibrium in case of a coordination failure among retailers
- In both these papers the entrant is passive when M bargains with retailers
- In BW98, incumbent M_a and entrant M_b simultaneously make offers to retailer R
- exclusion happens if and only if it is efficient for M_a, M_b and R jointly

Bernheim and Whinston (1998): Motivation (cont.)

- With sequential development of markets over time, it can happen that M_b does not enter while this would be efficient for M_a, M_b, R_1 and R_2 jointly
- reason is that markets are non-coincident: market 2 (and its retailer R_2) do not exist yet when M_a, M_b and R_1 bargain

Model

- bidding game: M_a and M_b make offers to R
- outside option for each player is 0
- Notation: (total) exclusive profits $\pi_a^e \geq \pi_b^e > 0$
- common profits: $0 < \pi^c < \pi_a^e + \pi_b^e$: M_a and M_b sell partial substitutes
- M_k demands net utility u_k for himself ($k = a, b$)
- timing of three stage game:
  - M_a and M_b simultaneously bid for representation at R; bid consists of net utility u_k^e for M_k in case of exclusive representation and u_k^c for common representation
  - retailer accepts compatible contracts (or no contract)
  - aggregate payoffs π_k^l materialize $k = a, b$; $l = c, e$ and R pays manufacturer(s) such that their demanded net payoff arises

Model (cont.)

- Hence R receives $u_R = \pi^c - u_a^c - u_b^c$ in case of common representation and $u_R = \pi^e - u_k^e$ in case of exclusive deal with M_k

Exclusive equilibria

- In exclusive equilibrium we have:
  - $u_a^c = u_b^c = +\infty$

$$\pi_a^e - u_a^e = \pi_b^e - u_b^e > 0 \tag{48}$$
$$u_a^e \geq 0 \geq u_b^e \tag{49}$$

  - R accepts offer from M_a
- To see why this is true, note that
  - if M_a sets $u_a^c = +\infty$, M_b's best response is to set $u_b^c = +\infty$ as well; hence exclusive equilibrium always exists
  - if $\pi_a^e - u_a^e < 0$, R should reject the offer
  - if $\pi_a^e - u_a^e = 0$, M_b can offer $u_b^e < \pi_b^e$ and win the contract
  - if $\pi_a^e - u_a^e < \pi_b^e - u_b^e$, R should accept M_b's offer
  - if $\pi_a^e - u_a^e > \pi_b^e - u_b^e$, M_a can raise u_a^e
  - if $u_a^e < 0$, M_a should withdraw offer

Exclusive equilibria (cont.)

  - if $u_b^e > 0$, M_b should reduce u_b^e slightly and win because of (48)
- There are multiple equilibria: best equilibrium for coalition M_a, M_b (making the offers) is $u_b^e = 0, u_a^e = \pi_a^e - \pi_b^e$ and $u_R = \pi_b^e$. M_a gets its contribution to total surplus
- if $\pi_a^e > \pi_b^e$ there also exist equilibria with $u_b^e < 0$ and $u_a^e < \pi_a^e - \pi_b^e$ but these are Pareto dominated for M_a and M_b by equilibrium with $u_b^e = 0$

Common equilibria

- If R serves both M_a and M_b we must have

$$\pi_a^e - u_a^e = \pi_b^e - u_b^e = \pi^c - u_a^c - u_b^c > 0 \tag{50}$$

and

$$u_k^c \geq 0 \tag{51}$$
$$u_k^e \leq u_k^c \tag{52}$$

for $k = a, b$

- To see why this is true, note that
    - if $\pi^c - u_a^c - u_b^c < 0$, R should reject the offer

Common equilibria (cont.)

- if $\pi^c - u_a^c - u_b^c = 0$ then $\pi_a^e + \pi_b^e - u_a^c - u_b^c > 0$, hence there is M_k such that $\pi_k^e - u_k^c > 0$; suppose k deviates by offering $\tilde{u}_k^e = u_k^c + \varepsilon$ with $\varepsilon > 0$. If R accepts the deviating offer, M_k clearly gains. By accepting, R earns

$$\pi_k^e - \tilde{u}_k^e = \pi_k^e - u_k^c - \varepsilon > 0 = \pi^c - u_a^c - u_b^c$$

  for $\varepsilon > 0$ small enough. Hence R accepts the deviating offer
- if $\pi_k^e - u_k^e > \pi^c - u_a^c - u_b^c$, R should have accepted M_k's exclusive offer
- if $\pi_k^e - u_k^e < \pi^c - u_a^c - u_b^c$, M_{-k} can profitably deviate by slightly increasing u_{-k}^c and setting $u_{-k}^e = +\infty$
- if $u_k^c < 0$, M_k should withdraw his offer
- if $u_k^e > u_k^c \geq 0$, M_k should reduce u_k^e slightly and win exclusive contract because of (50)

Common equilibria (cont.)

- common equilibrium exists if and only if $\pi^c \geq \pi_a^e$. Two ways to see this:
  - Write equation (50) as
    $$\pi^c = \pi_a^e + [u_a^c - u_a^e] + u_b^c \geq \pi_a^e$$
    because $u_a^c \geq u_a^e$ and $u_b^c \geq 0$
  - suppose $\pi^c < \pi_a^e$, coalition R and M_a can deviate from common outcome and earn $\pi_a^e > \pi^c - u_b^c$
- multiple common equilibria: the lower u_a^e, u_b^e the worse the equilibrium is for M_a, M_b; in the light of (52), the best manufacturers can do is to set $u_k^e = u_k^c = u_k$
  - to see this, suppose M_k chooses $u_k^e < u_k^c$
  - Then equation (50) becomes
    $$\pi_k^e - u_k^c < \pi_k^e - u_k^e = \pi^c - u_k^c - u_{-k}^c$$

Common equilibria (cont.)

- hence $u^c_{-k} < \pi^c - \pi^e_k$ while actually both manufacturers can get $u^c_k = \pi^c - \pi^e_{-k}$ (see below); hence the coalition of M_a, M_b is better off by setting $u^e_k = u^c_k$

- with $u^c_k = u^e_k = u_k$ we solve two equations in two unknowns:

$$\begin{cases} \pi^e_a - u_a & = \pi^c - u_a - u_b \\ \pi^e_b - u_b & = \pi^c - u_a - u_b \end{cases}$$

  solution is given by

$$\begin{aligned} u_a &= \pi^c - \pi^e_b \\ u_b &= \pi^c - \pi^e_a \end{aligned}$$

  each manufacturer gets marginal contribution to surplus π^c

- R earns

$$u_R = \pi^c - u_a - u_b = \pi^e_a + \pi^e_b - \pi^c > 0 \qquad (53)$$

Common vs Exclusive: three cases

1. if $\pi^c < \pi_a^e$, there only exists an exclusive equilibrium with M_a, payoffs are

$$u_a^e = \pi_a^e - \pi_b^e \tag{54}$$
$$u_b^e = 0 \tag{55}$$
$$u_R^e = \pi_b^e \tag{56}$$

2. if $\pi^c > \pi_a^e$ then
   - exclusive equilibrium above still exists
   - common equilibrium exists as well, payoffs:

$$u_a^c = \pi^c - \pi_b^e > \pi_a^e - \pi_b^e = u_a^e \tag{57}$$
$$u_b^c = \pi^c - \pi_a^e > 0 = u_b^e \tag{58}$$
$$u_R^c = \pi_a^e + \pi_b^e - \pi^c < \pi_b^e = u_R^e \tag{59}$$

because $\pi^c > \pi_a^e$

Common vs Exclusive: three cases (cont.)

- hence M_a, M_b are better off in common outcome while R is better off in exclusive outcome. However, M_a, M_b make the offers and hence should be able to coordinate on their preferred outcome

3. if $\pi^c = \pi_a^e$, there is a unique Pareto dominant payoff vector (for M_a, M_b) which can be achieved through either exclusive or common representation

Two principles

1. M_a, M_b by choosing the form of representation that is most profitable for them, maximize the joint surplus of M_a, M_b and R
2. Exclusion can only happen if there are contracting externalities
    - let $\bar{\pi}^c$ denote the joint profit when M_a, M_b fully cooperate
    - $\bar{\pi}^c \geq \pi_a^e$ when M_a, M_b cooperate, they can always replicate the exclusive outcome
    - Hence $\pi_a^e > \pi^c$ and exclusion can only happen if $\bar{\pi}^c > \pi^c$: there are contracting externalities, e.g.
        - HT90 type of problem on the supply side: input suppliers of M_a, M_b produce with costs $C(x_a + x_b)$ with $C', C'' > 0$
        - incentive contracts provided by M_a, M_b for a risk averse R: section V in the paper

Non-coincident markets

- two retail markets develop sequentially over time:
  - first period/market M_a, M_b and R_1 bargain
  - second period/market M_a, M_b and R_2 bargain
- M_a is incumbent and M_b is entrant who needs to cover entry cost $f > 0$
- entry by M_b is only profitable if M_b can serve both R_1 and R_2
- in the first period, M_a, M_b cannot bargain with R_2 because this market does not exist yet
- for each market t:
  - x^*_{ta}, x^*_{tb} solve

$$\pi^c_t = \bar{\pi}^c_t = \max_{x_a, x_b} \{R_t(x_a, x_b) - c_a x_a - c_b x_b\}$$

Non-coincident markets (cont.)

- x^e_{tk} solves
$$\pi^e_{tk} = \max_{x_k}\{R_t(x_k, 0) - c_k x_k\}$$
- with $\pi^e_{ta} + \pi^e_{tb} > \pi^c_t$
- hence if markets would be completely separate, common outcome prevails because $\pi^c_t = \bar{\pi}^c_t \geq \pi^e_{tk}$

- Three assumptions:
  - C1 $0 < \pi^c_2 - \pi^e_{2a} < f$: if M_b only sells to R_2, M_2's contribution to total profits does not cover entry cost f
  - C2 $\pi^c_1 - \pi^e_{1a} + \pi^c_2 - \pi^e_{2a} > f$: M_2's contribution to total profits on both markets does cover f: aggregate profits (for $R_{1,2}, M_{a,b}$) maximized if M_b enters

Non-coincident markets (cont.)

C3 $\pi^e_{1a} + \pi^e_{2a} > \pi^c_1 + [\pi^c_2 - \pi^e_{2a}] + [\pi^c_2 - \pi^e_{2b}] - f$: profits of M_a, M_b and R_1 higher if M_b does not enter

Proposition 10

If (C3) holds, all undominated equilibria involve effective exclusion of M_b from market 1 (and hence from market 2)

- **Proof.** If M_b enters, M_a, M_b and R_1 get joint profit

$$\pi^c_1 + [\pi^c_2 - \pi^e_{2a}] + [\pi^c_2 - \pi^e_{2b}] - f$$

- if M_b does not enter, their joint profit is

$$\pi^e_{1a} + \pi^e_{2a}$$

Non-coincident markets (cont.)

- by (C3) the latter exceeds the former. □
- by excluding M_b, the bargaining power of M_a is increased vs R_2 which maximizes the joint profit of M_a, M_b and R_1

Explicit exclusion clauses

- In an exclusive outcome, do M_a and R_1 need an explicit exclusionary clause in their contract?
- In other words, once the contract between M_a and R_1 is signed, do M_b and R_1 have an incentive to deviate (if there is no such clause)?
- joint profit of M_b and R_1 is higher if they stick to the agreement than if they deviate if and only if

$$R_1(x_{1a}^e, 0) \geq R_1(x_{1a}^e, x_{1b}) - c_b x_{1b} + [\pi_2^c - \pi_{2a}^e - f] \text{ for all } x_{1b} > 0 \tag{60}$$

- if (60) does hold, M_a and R_1 have effective exclusion without an explicit clause in their contract

Explicit exclusion clauses (cont.)

- if (60) does not hold, M_a and R_1 need an explicit exclusionary clause in their contract
- if (60) does not hold, and explicit exclusionary clauses are forbidden by law, does his imply the common outcome?
- Not if the following inequality holds:

$$\max_{x_{a1}^e \text{ s.t } (60)} \{R_1(x_{a1}^e, 0) - c_a x_{a1}^e\} + \pi_{2a}^e > \pi_1^c + [\pi_2^c - \pi_{2b}^e] + [\pi_2^c - \pi_{2a}^e] - f$$

Summary

- BW98: bidding game; no beliefs
- manufacturers choosing the form of representation that maximizes their profits, maximize joint surplus
- exclusion can only happen if there are contracting externalities such that common (total) profits are below common profits that cooperating manufacturers could achieve
- with non-coincident markets exclusion can be profitable for all parties in the first market because it raises their bargaining power in the second market

Part VI

Details of the bargaining environment

Motivation

- We consider the case with one upstream M and N downstream retailers R_i where M makes the offers (offer game)
- What is the effect of public offers vs private offers?
- In HT90 private offers lead to inefficiently high output and public offers lead to an efficient outcome
- in Katz and Shapiro (1986a,b) and Kahn and Mookerjee (1998) there are public offers and too much trade
- in Grossman and Hart (1980) there is a trader with public offers and too little trade
- what is causing these differences?
- How do externalities when trading differ from externalities when retailers do not trade?

Motivation (cont.)

- what is the effect if the manufacturer cannot discriminate in its offers to retailers?
- what happens if M wants to have unique implementation (like in Winter (2004))?

Inefficiency due to externality on non-trader

- M's trade with R_i is denoted x_i; vector of such trades x
- $X = \sum_{i \in N} x_i$ and $X_{-i} = \sum_{j \neq i} x_j$
- R_i's payoff: $u_i(x) - t_i$; usually $u_i(x) = p(X)x_i$
- M's payoffs: $u_m(x) = \sum_{i \in N} t_i - c(X)$
- default ("no trade") point: $x_i = 0$; R_i's payoff: $u_i(0, x_{-i})$
- In HT90 $u_i(0, x_{-i}) = 0$: if retailer does not buy from M, he does not produce
- In Katz and Shapiro (1986a) profit of firm that does not buy license depends on how many firms do buy a license: $L(k)$ with $L'(k) \leq 0$.

Inefficiency due to externality on non-trader (cont.)

- set of efficient trades:

$$\xi^* = \arg\max_x \sum_{i \in N} u_i(x) - c(X) \tag{61}$$

- Two-stage game:
  1. M commits to set $\{(x_i, t_i)\}_{i \in N}$ of publicly observable bilateral contract offers to retailers
  2. retailers simultaneously decide whether to accept or reject their resp. offers
- IR-constraints for retailers:

$$u_i(x) - t_i \geq u_i(0, x_{-i}) \tag{62}$$

for all $i \in N$

Inefficiency due to externality on non-trader (cont.)

- As there is no reason for M to leave rents to retailers, we can solve for t_i
- M's set of profit maximizing trades:

$$\xi^{pu} = \arg\max_x \sum_{i \in N} [u_i(x) - u_i(0, x_{-i})] - c(X) \qquad (63)$$

Proposition 11

If $u_i(0, x_{-i})$ does not depend on x_{-i} for all i, then $\xi^* = \xi^{pu}$.

- in this case the optimization problems (61) and (63) coincide.
- when externalities on non-traders are absent and M commits to compensate retailers for externalities imposed upon them, M internalizes the externalities and implements efficient trades

Inefficiency due to externality on non-trader (cont.)

- This explains why HT90 find an efficient outcome with public contracts
- if we do get an inefficient outcome, in which direction does it go?
- Define

$$\Xi^* = \{\sum_i x_i | x \in \xi^*\} \tag{64}$$

$$\Xi^{pu} = \{\sum_i x_i | x \in \xi^{pu}\} \tag{65}$$

Proposition 12

With negative externalities on non-traders ($\partial u_i(0, x_{-i})/\partial x_j < 0$ for $j \neq i$), we find $\Xi^{pu} \geq \Xi^*$.

Inefficiency due to externality on non-trader (cont.)

- **Proof.** Define the following functions

$$W(X) = P(X)X - c(X) \tag{66}$$

$$R(X) = \min_{x}\{\sum_i u_i(0, x_{-i}) | \sum_i x_i = X\} \tag{67}$$

- Then R is non-increasing in X:
  - consider X, X' with $X' \geq X$
  - take x such that $\sum_i x_i = X$ and $R(X) = \sum_i u_i(0, x_{-i})$
  - then there exists x' such that $x'_i \geq x_i$ for all $i \in N$ and $\sum_i x'_i = X'$

Inefficiency due to externality on non-trader (cont.)

- because of negative externalities on non-traders, we have

$$u_i(0, x_{-i}) \geq u_i(0, x'_{-i})$$

and therefore

$$R(X') \leq \sum_i u_i(0, x'_{-i}) \leq \sum_i u_i(0, x_{-i}) = R(X)$$

- to simplify, assume that W and R are differentiable and compare the first order conditions for the optimization problems $\max_X W(X)$ and $\max_X W(X) - R(X)$:

$$\begin{aligned} W'(X^*) &= 0 \\ W'(X^{pu}) &= R'(X^{pu}) \leq 0 \end{aligned}$$

- hence $\Xi^* \leq \Xi^{pu}$. $\square$

Inefficiency due to externality on non-trader (cont.)

- Hence with negative externalities on non-traders we get "overproduction": M sells more than the monopoly output level in order to worsen retailers' outside option
- In Katz and Shapiro (1986a) there is a negative externality of a trader (downstream firm that buys license) on a non-trader (firm that works with old technology); hence R&D lab tends to sell too many licenses
- In Katz and Shapiro (1986b) there is a negative externality of people that buy sponsored technology B on people that use non-sponsored technology A; firm B induces too many people (at $t = 1$) to buy its technology; hence B survives although it is inferior to A
- in Grossman and Hart (1980) there is positive externality of shareholders tendering to raider on shareholders that do not tender and hence there is too little trade (proposition 12 with positive externalities on non-traders)

Private offers: externality on efficient traders

- similar two-stage game as above, but now M makes private offer to each R_i
- In other words, M cannot commit to abstain from renegotiation with R_i after other retailers have accepted their contract
- We assume that retailers have passive beliefs: if $\hat{x}$ is the equilibrium trade and R_i receives a deviating offer, he still beliefs the other retailers were offered $\hat{x}_{-i}$
- Hence the IR constraint can now be written as

$$u_i(x_i, \hat{x}_{-i}) - t_i \geq u_i(0, \hat{x}_{-i}) \tag{68}$$

Private offers: externality on efficient traders (cont.)

- As above we can solve for t_i:

$$t_i = u_i(x_i, \hat{x}_{-i}) - u_i(0, \hat{x}_{-i})$$

- Note that when making offers x_i to retailers, M takes $\hat{x}_{-i}$ as given
- Hence $\hat{x}$ is an equilibrium trade if and only if

$$\hat{x} \in \arg\max_{x} \sum_i u_i(x_i, \hat{x}_{-i}) - c(X) \tag{69}$$

- let ξ^{pr} denote the set of $\hat{x}$ that satisfy equation (69)

Proposition 13

If there exists $x^* \in \xi^*$ such that $u_i(x_i^*, x_{-i})$ does not depend on x_{-i} for all $i \in N$, then $\xi^{pr} \subset \xi^*$

Private offers: externality on efficient traders (cont.)

- **Proof.** For any $x^{pr} \in \xi^{pr}$ equation (69) implies that

$$\sum_i u_i(x^{pr}) - c(X^{pr}) \geq \sum_i u_i(x_i^*, x_{-i}^{pr}) - c(X^*) = \sum_i u_i(x^*) - c(X^*)$$

  and hence $x^{pr} \in \xi^*$. □

- To see which way the direction of the distortion goes, define:
  - externalities on efficient traders are negative if for all $x^* \in \xi^*$ and each $i \in N$, $u_i(x_i^*, x_{-i})$ is non-increasing in x_{-i}. This is true if $u_i(x_i, x_{-i}) = p(X)x_i$.
  - $\Xi^{pr} = \{\sum_i x_i | x \in \xi^{pr}\}$

Proposition 14

Assume $u_i(x_i, x_{-i}) = p(X)x_i$. *Then* $\Xi^{pr} \geq \Xi^*$

Private offers: externality on efficient traders (cont.)

- **Proof.** First order condition for $x^{pr} \in \xi^{pr}$ can be written as

$$\frac{\partial u_i}{\partial x_i}(x_i^{pr}, x_{-i}^{pr}) - c'(X^{pr}) = 0$$

- FOC for $x^* \in \xi^*$:

$$\frac{\partial u_i}{\partial x_i}(x_i^*, x_{-i}^*) - c'(X^*) = -\sum_{j \neq i} \frac{\partial u_j}{\partial x_i}(x_j^*, x_{-j}^*) \geq 0$$

- hence we find $X^* \leq X^{pr}$. $\square$

Comparing public and private offers

- Assume $u(x_i, x_{-i}) = p(X)x_i$ and $u(0, x_{-i}) = u_0(X_{-i})$

Proposition 15

If $p'(X)X < nu_0'(X_{-i})$ for each X, then $X^{pu} < X^{pr}$.

- **Proof.** *Check* that first order conditions for x_i^{pr}, x_i^{pu} can be written as resp.

$$p'(X^{pr})X^{pr} + p(X^{pr}) - c'(X^{pr}) = p'(X^{pr})X_{-i}^{pr}$$
$$p'(X^{pu})X^{pu} + p(X^{pu}) - c'(X^{pu}) = (n-1)u_0'(X_{-i})$$

- by writing the RHS of the first equation as $(n-1)p'(X^{pr})X^{pr}/n$, the result follows. $\square$

Overview

- S99 considers private vs public offers
- here we only consider public offers
- as above, contract with i cannot depend on the choice made by $j \neq i$
- we consider the cases where M can and cannot discriminate between retailers
- if M cannot discriminate, she has to offer each retailer the same contract
- further, we compare the difference between simple and unique implementation
- In S99 we considered the case where M can *d*iscriminate and only looked at *s*imple implementation. We denote the set of outcomes defined in equation (63) now by ξ_d^s

Overview (cont.)

- below we will characterize the sets ξ_n^s and ξ_d^u
- section 4.1.2 in the paper characterizes ξ_n^u
- characterizing ξ_d^u, ξ_n^u turns out to be straightforward for the case of decreasing externalities (see below)

Model

- $u_i(x_i, x_{-i}) = p(x_i + X_{-i})x_i$
- hence we focus on negative externalities: $\partial u_i / \partial x_j = p'(X)x_i < 0$ for each $j \neq i$
- externalities can be either
  - strictly increasing: $\partial^2 u_i / \partial x_i \partial x_j = p''(X)x_i + p'(X) > 0$ or
  - strictly decreasing: $\partial^2 u_i / \partial x_i \partial x_j = p''(X)x_i + p'(X) < 0$
- as above $\Xi^* = \arg\max_X P(X)X - c(X)$
- proposition 12 above: with negative externalities at $(0, \ldots, 0)$ we have $\Xi^* \leq \Xi_d^s$

Non-discrimination and simple implementation

- M offers the same menu of contracts to all retailers:

$$S = \{(0,0)\} \cup \{(x_i, t_i) | i \in N\} \tag{70}$$

  where R_i is supposed to choose (x_i, t_i)
- in equilibrium:
  - no R_i prefers (0,0) above (x_i, t_i):

$$u(x_i, X_{-i}) - t_i \geq u(0, X_{-i}) \tag{IR_i}$$

  - R_i prefers (x_i, t_i) above (x_j, t_j) for $j \neq i$:

$$u(x_i, X_{-i}) - t_i \geq u(x_j, X_{-i}) - t_j \tag{IC_{ij}}$$

Non-discrimination and simple implementation (cont.)

- hence different retailers choose different contracts if and only if they have different expectations about what others do: X_{-i} becomes their type
- with decreasing externalities, retailer with higher type X_{-i} has a lower incentive to trade

Increasing externalities

Lemma 6

With strictly increasing externalities, each R_i chooses the same contract $(\bar{x}, \bar{t})$ from S. $\bar{x}, \bar{t} > 0$ if and only if

$$u(\bar{x}, \ldots, \bar{x}) - \bar{t} \geq u(0, \bar{x}, \ldots, \bar{x})$$

- **Proof.** Suppose not, i.e. suppose $x_i > x_j$ for some $R_i, R_j \in N$. But then R_j's type $X_{-j} = x_i + X_{-i-j}$ is higher than R_i's type $X_{-i} = x_j + X_{-i-j}$ which makes higher trade relatively more attractive to R_j. Hence R_j strictly prefers x_i, which is a contradiction. $\square$

Increasing externalities (cont.)

- to compare the outcome with non-discrimination with the efficient outcome, we define welfare when planner has to choose symmetric outcomes:

$$\bar{W}(X) = Nu(X/N, \dots, X/N) - c(X) \tag{71}$$

- set of efficient symmetric trades:

$$\bar{\xi}^* = \arg\max_X \bar{W}(X) \tag{72}$$

- equilibrium outcomes:

$$\xi_n^s = \arg\max_x p(Nx)Nx - c(Nx) - Nu(0, (N-1)x) \tag{73}$$

- *Check* that the following claim is correct:

Proposition 16

If $\partial u(0, X_{-i})/\partial X_{-i} < 0$ *then* $\xi_n^s \geq \bar{\xi}^*$

Decreasing externalities

- with decreasing externalities, M can implement equilibrium in which ex ante identical retailers choose different contracts from the same menu S
- without loss of generality, order retailers such that $x_1 \leq x_2 \leq \ldots \leq x_N$ in M's profit maximizing outcome
- this implies $X_{-1} \geq X_{-2} \geq \ldots \geq X_{-N}$

Lemma 7

(IR_1) and $(IC_{i,i-1})$ for $i \in \{2, \ldots, N\}$ are binding; the other incentive compatibility and individual rationality constraints can be ignored.

Decreasing externalities (cont.)

- **Proof.** Suppose $(IC_{2,1}), (IC_{3,2})$ holds. *Check* that this implies

$$t_3 - t_1 \leq u(x_2, X_{-2}) + u(x_3, X_{-3}) - [u(x_1, X_{-2}) + u(x_2, X_{-3})]$$

- Suppose by contradiction that $(IC_{3,1})$ does not hold:

$$u(x_3, X_{-3}) - u(x_1, X_{-3}) < t_3 - t_1$$

- Adding these two inequalities yields

$$u(x_2, X_{-3}) - u(x_1, X_{-3}) < u(x_2, X_{-2}) - u(x_1, X_{-2})$$

- or equivalently

$$\int_{x_1}^{x_2} \frac{\partial u}{\partial x}(x, X_{-3})dx < \int_{x_1}^{x_2} \frac{\partial u}{\partial x}(x, X_{-2})dx$$

Decreasing externalities (cont.)

- but $X_{-3} \leq X_{-2}$ and decreasing externalities imply

$$\frac{\partial u}{\partial x}(x, X_{-3}) \geq \frac{\partial u}{\partial x}(x, X_{-2})$$

- which is a contradiction.
- Next we show that

$$u(x_i, X_{-i}) - t_i > u(0, X_{-i}) \tag{74}$$

for $i = 2, \ldots, N$

- Suppose not, that is suppose

$$u(x_i, X_{-i}) - t_i \leq u(0, X_{-i})$$

Decreasing externalities (cont.)

- add this inequality to $(IC_{i,1})$ with $t_1 \leq u(x_1, X_{-1}) - u(0, X_{-1})$, this yields

$$u(x_1, X_{-i}) - u(0, X_{-i}) \leq u(x_1, X_{-1}) - u(0, X_{-1})$$

writing both sides as an integral from 0 to x_1 we get a contradiction, as above. $\square$

- when M can discriminate, each IR holds with equality
- hence equation (74) implies that M strictly looses from the fact he cannot discriminate
- in general it is hard to compare ξ^* and ξ_n^s as there is a combination of positive/negative externalities and decreasing externalities
- To illustrate

$$t_2 = u(x_2, X_{-2}) - u(x_1, X_{-2}) + [u(x_1, X_{-1}) - u(0, X_{-1})]$$

Decreasing externalities

- with decreasing externalities, unique implementation does not lead to extra costs for M
- First consider the case with *discrimination*: M offers R_i menu $\{(0,0),(x_i,t_i)\}$
- take an outcome from ξ_d^s and reduce each t_i slightly such that

$$u(x_i, X_{-i}) - t_i > u(0, X_{-i})$$

- then given that retailers R_{-i} play equilibrium, R_i prefers (x_i, t_i) strictly above $(0,0)$
- to see uniqueness: suppose $k \leq N-1$ retailers would switch to $x_k' = (0,0)$, then R_i faces $X_{-i}' < X_{-i}$

Decreasing externalities (cont.)

- decreasing externalities imply

$$u(x_i, X'_{-i}) - u(0, X'_{-i}) > u(x_i, X_{-i}) - u(0, X_{-i}) > t_i$$

  that is, (x_i, t_i) becomes even more attractive
- this is the main difference with increasing externalities where retailers $k \neq i$ switching to $(0, 0)$, makes $(0, 0)$ more attractive for R_i
- *non-discrimination* with decreasing externalities allows M to implement an outcome that is asymmetric ex post
- unique implementation is impossible in a sense that it is irrelevant to M:
  - it does not matter whether R_1 chooses (x_1, t_1) and R_2 chooses (x_2, t_2) or
  - R_2 chooses (x_1, t_1) and R_1 chooses (x_2, t_2)

Unique implementation with increasing externalities

- with increasing externalities unique implementation is harder: if X_{-i} increases, it becomes more attractive for R_i to increase x_i as well, leading to multiple equilibria
- M needs to compensate each retailer for each step that he takes from initial starting point to the desired equilibrium
- we only illustrate this with an example (see S03a section 4.1.1. for more details and a comparison of ξ^* and ξ_d^u in proposition 5)
- consider the following payoff matrix for R_1 and R_2; M has payoff zero unless both R_1 and R_2 choose $x_1 = x_2 = 30$ in which case $u_M = 10$

Unique implementation with increasing externalities (cont.)

		R_2		
	x_i	10	20	30
	10	8,8	4,4	0,0
R_1	20	4,4	2,2	1,1
	30	0,0	1,1	0,0

- unique Nash equilibrium in the subgame of $R_{1,2}$ is $x_1 = x_2 = 10$
- M would like to implement $x_1 = x_2 = 30$ in a unique way, what should he offer $R_{1,2}$?
- If uniqueness is not an issue, M offers both retailers $(x_i, t_i) = (30, 1)$
- to get uniqueness, use "round-robin optimization" with $\varepsilon > 0$ but small:

Unique implementation with increasing externalities (cont.)

1. offer R_1: $(20, 4 + \varepsilon)$ to move him from $x = 10$ to $x = 20$
2. offer R_2: $(20, 2 + \varepsilon)$, we get:

	x_i	R_2: 10	20	30
	10	8,8	4,6+ε	0,0
R_1	20	8+ε,4	6+ε,4+ε	5+ε,1
	30	0,0	1,3+ε	0,0

3. offer R_1: $(30, 5 + 2\varepsilon)$ to move him from $x = 20$ to $x = 30$
4. offer R_2: $(30, 3 + 2\varepsilon)$, we get:

	x_i	R_2: 10	20	30
	10	8,8	4,6+ε	0,3+2ε
R_1	20	8+ε,4	6+ε,4+ε	5+ε,4+2ε
	30	5+2ε,0	6+2ε,3+ε	5+2ε,3+2ε

Unique implementation with increasing externalities (cont.)

- Hence unique equilibrium is for both retailers to choose $x = 30$ and R_1 gets $t_1 = 5 + 2\varepsilon$ while R_2 gets $t_2 = 3 + 2\varepsilon$

Summary

- Both S99 and S03a are offer games
- with negative externalities on non-traders, public discriminatory offers lead to oversupply to reduce retailers' outside options
- with private offers and negative externalities on traders, there is oversupply because passive beliefs cause M to ignore the reduction in his transfers due to the externality
- if externalities on traders are smaller than on non-traders, private offers lead to a bigger oversupply
- if offers cannot discriminate (and simple implementation):
  - with increasing externalities:
    - all retailers choose the same contract
    - with negative externalities on non-traders, there is oversupply compared to efficient symmetric outcome
  - with decreasing externalities:

Summary (cont.)

- ex post asymmetric outcome is possible
- non-discrimination leads to strict loss for manufacturer as *IC*-constraints leave rents to retailers (in contrast to *IR*-constraints)
- unique implementation does not raise costs for M

- with unique implementation, discriminatory offers and increasing externalities:
  - if $t_i = x_i = 0$ does not lead to manufacturer's preferred outcome
  - M needs to bring retailers "step-by-step" to the desired equilibrium outcome
  - retailers need to be compensated for each step that they take

Part VII

Two (and more) parties on both sides

Motivation

- BW98-method to find which contractual arrangements can form an equilibrium and how the surplus is distributed between parties works fine with two upstream firms and one downstream firm
- they do this in the form of a bidding game so that beliefs play no role
- if there would be two downstream firms as well, the situation would be substantially more complicated:
  - R_1 needs to form beliefs about the offers made to R_2
  - if M_a has an exclusive contract with R_1, R_1's surplus equals $\pi_1^{(ae)}$, this can take 3 different values depending on whether
    1. M_a has an exclusive contract with R_2
    2. M_b has an exclusive contract with R_2
    3. M_a and M_b have a common contract with R_2

Motivation (cont.)

- deviations become more complicated: suppose M_a has an exclusive contract with R_1 and a common contract with R_2, possible deviations for M_a include (beside changing one contract at a time):
  - deviating to a common contract with R_1 and an exclusive contract with R_2
  - dropping the contract with R_1 and switching to an exclusive contract with R_2
- This makes it less tractable and hence less attractive
- Segal (2003b) uses cooperative game theory to analyze exclusion (as well as inclusion and collusion) contracts and their effects on the payoff distribution

Cooperative game theory

- set of players $N = \{1, 2, \ldots, n\}$
- characteristic function $v : 2^N \to \mathbb{R}$ where $v(S)$ is worth/value of coalition $S \subset N$, $v(\emptyset) = 0$
- Note: 2^N because each player $1, 2, \ldots, N$ can be either "in" or "out" of a coalition S
- agents own resources that can be combined to generate surplus
- Notation

$$[\Delta_i v](S) = v(S \cup i) - v(S \setminus i) \tag{75}$$

$$[\Delta^2_{ij} v](S) = \Delta_i[\Delta_j v](S) = v(S \cup i \cup j) + v(S \setminus i \setminus j) - [v(S \setminus j \cup i) + v(s \setminus i \cup j)] \tag{76}$$

$$[\Delta^3_{ijk} v] = \Delta_i[\Delta^2_{jk} v] \tag{77}$$

Cooperative game theory (cont.)

- $\Delta_i v$: i's marginal contribution to coalition S
- $\Delta^2_{ij} v$: i's effect on j's marginal contribution to coalition S:
  - if $\Delta^2_{ij} v > 0$ for all i, j, S, then coming late in the ordering (see below) yields a higher contribution to the surplus: i is complementary to j
  - if $\Delta^2_{ij} v < 0$ for all i, j, S: i is substitutable to j
- $\Delta^3_{ijk} v$: effect of player k on the complementarity between i and j

Solution concept

- The idea is that each player gets his "expected" marginal contribution to the total surplus
- Let Π denote set of orderings (group of permutations) of N
- $\pi \in \Pi$ denotes a particular ordering
- $\pi(i)$ denotes the rank of player $i \in N$ in ordering π
- $\pi^i = \{j \in N | \pi(j) \leq \pi(i)\}$: set of players that come before i in ordering π
- In a particular ordering π, i's marginal contribution (at the moment of entry) equals $\Delta_i v(\pi^i)$
- i's "ex ante" value equals its expected marginal contribution where the expectation is taken over all orderings
- formally, $P(\Pi) = \{\alpha \in \mathbb{R}_+^{\Pi} | \sum_{\pi \in \Pi} \alpha_\pi = 1\}$ denotes the set of probability distributions over Π

Solution concept (cont.)

- $\alpha \in P(\Pi)$ gives rise to i's ex ante payoff

$$f_i^\alpha = \sum_{\pi \in \Pi} \alpha_\pi \Delta_i v(\pi^i)$$

- joint (ex ante) value of a group M of players: $f_M^\alpha(v) = \sum_{i \in M} f_i^\alpha(v)$

Fundamentalist interpretation

- We first (ex ante) agree that each player i gets f_i^α; if everyone agrees this is distribution of payoffs is implemented
- if someone objects, we play the following game:
  - nature draws an order π from Π (according to probability distribution α)
  - re-lable players such that $\pi(i) = i$
  - player 1 enters first and gets $v(\{1\})$
  - player 2 enters and proposes a distribution of surplus between 1 and 2, if 1 agrees this is what 2 gets, if 1 disagrees 2 is "out" and player 3 enters
  - player 2 will claim $v(\{1, 2\}) - v(\{1\})$ (his marginal contribution) as his own payoff and 1 accepts
  - player $k + 1$ ($k \geq 2$) enters and proposes a distribution of payoffs; if someone in coalition $\{1, 2, \ldots, k\}$ disagrees, k is out, otherwise k gets what he claimed

Fundamentalist interpretation (cont.)

  - each player k claims his marginal contribution and leaves the distribution of payoffs for π^k unchanged

- in expected terms this gives everyone his ex ante value f_i^α

Defining the contracts

- *exclusive contract* gives player i the right to exclude j to deal with others (till i arrives):
  - has no effect on orderings where i arrives before j (if j contributes to the surplus of the existing coalition, there is no reason for i to stop this)
  - if j arrives before i, j cannot deal with the existing coalition until i arrives; i's marginal contribution upon arrival then includes j's contribution to the then (bigger) coalition
  - players arriving between j and i cannot claim their marginal surplus generated by interaction with j
- *inclusive contract* gives i the right to use j's resource himself (if j has not arrived yet):
  - has no effect if j arrives before i

Defining the contracts (cont.)

- if i arrives before j, i introduces his own resource and that of j and claims j marginal contribution now with the smaller coalition
- players arriving between i and j claim their marginal surplus generated by interaction with j's resource

- *collusive contract* gives full control of both players' resources to proxy player i (j is dummy)
    - this is the "sum" of exclusive and inclusive contract

Access structure

- before this cooperative bargaining starts, players can agree on contracts
- they can use lump sum transfers such that everyone involved in the contract gains from it
- assumption: contracts do not affect the bargaining procedure
- let $A(S) \subset N$ denote the set of agents whose resources are available to coalition S
- e.g. if i and j have an exclusive contract and j has arrived before i then the coalition π^j does not have access to j's resources
- if i and j have an inclusive contract and i has arrived before j then the coalition π^i does have access to j's resources

Access structure (cont.)

- access structure A affects the characteristic function:

$$vA(S) \equiv [vA](S) = v(A(S)) \tag{78}$$

- incentive for coalition M to enter into contract A is determined by the comparison of $f_M^\alpha(v)$ and $f_M^\alpha(vA)$
- Segal only considers contracts A that do not affect total surplus $v(N)$:

$$f_M^\alpha(vA) + f_{N\setminus M}^\alpha(vA) = v(N) = f_M^\alpha(v) + f_{N\setminus M}^\alpha(v) \tag{79}$$

- from this it follows that a contract is profitable to coalition M if and only if it imposes negative externalities on coalition $N \setminus M$

Exclusion

- consider exclusive contract E_i^j that gives player i the right to exclude j (in the sense defined above)
- access structure:

$$E_i^j(S) = \begin{cases} S & \text{if } i \in S \\ S \setminus j & \text{otherwise} \end{cases}$$

- because of equation (79), this can only be profitable if leads to negative external effects:
  - if $\pi(i) < \pi(j)$ (i arrives before j) there is no effect
  - agents arriving before j or after i are not affected
  - only players k arriving after j and before i $(\pi(j) < \pi(k) < \pi(i))$ see their marginal contribution reduced by

$$\Delta_k v(\pi^k) - \Delta_k(\pi^k \setminus j) = \Delta_{jk}^2 v(\pi^k)$$

Exclusion (cont.)

- taking expectations over orderings, we find the effect on k's payoffs:

$$f_k^{\alpha}(vE_i^j) - f_k^{\alpha}(v) = -\sum_{\pi \in \Pi | \pi(j) < \pi(k) < \pi(i)} \alpha_\pi \Delta_{jk}^2 v(\pi^k) \quad (80)$$

- note that i is never a member of the coalitions on the RHS of this equation
- if j is complementary to every player $k \neq i$ ($\Delta_{jk}^2 v > 0$) in the absence of i, the change in k's payoffs is negative
- hence the contract E_i^j is profitable for i and j (and i can compensate j ex ante for accepting such a contract)
- because players k with $\pi(j) < \pi(k) < \pi(i)$ have no access to j's resources:

Exclusion (cont.)

  - their marginal contribution to the surplus (at the moment they arrive) is reduced (in case of complements)
  - hence they can only claim a smaller share of the cake
  - when i arrives, j's resources become available to everyone in π^i, the total cake equals $v(\pi^i)$, the complementarity between j and k now adds to i's marginal contribution to the surplus
  - hence i can claim a bigger share of the cake $v(\pi^i)$

- note that "by the end of the day" everyone has access to j's resources
- hence exclusion does not affect total surplus (see (79)) only the distribution of surplus

Inclusion

- consider inclusive contract I_i^j that gives i the right to bring in j's resources (if i arrives before j):

$$I_i^j(S) = \begin{cases} S & \text{if } i \notin S \\ S \cup j & \text{otherwise} \end{cases}$$

- consider the external effects on a player k:
  - if $\pi(j) < \pi(i)$ there is no effect
  - agents arriving before i or after j are not affected
  - only players k arriving after i and before j see their marginal contribution increased by

$$\Delta_k v(\pi^k \cup j) - \Delta_k(\pi^k) = \Delta^2_{jk} v(\pi^k)$$

Inclusion (cont.)

- taking expectations over orderings, we find the effect on k's payoffs:

$$f_k^{\alpha}(vI_i^j) - f_k^{\alpha}(v) = \sum_{\pi \in \Pi | \pi(i) < \pi(k) < \pi(j)} \alpha_\pi \Delta_{jk}^2 v(\pi^k)$$

- note that i is always a member of the coalition on the RHS of this equation
- if j is complementary to each player $k \neq i$ in the presence of i, the inclusive contract I_i^j has a positive externality on k: when k arrives he now has access to j's resources which increases his marginal contribution to the surplus $v(\pi^k)$
- hence with complementarity, the contract I_i^j is not profitable for i and j and will not be signed in equilibrium
- a sufficient condition for I_i^j to be profitable is that j is substitutable to each player $k \neq i$ in the presence of i

collusion

- some people have the intuition that being "bigger" leads to more bargaining power
- this is not always true; see, for instance, the merger paradox in Cournot markets
- What property in the payoff structure affects whether collusion is profitable or not?
- It depends on the sign of $\Delta^3_{ijk} v$
- collusion is seen here as the combination of inclusion and exclusion:

$$C^j_i(S) = \begin{cases} S \cup j & \text{if } i \in S \\ S \setminus j & \text{otherwise} \end{cases}$$

- this is a rather formal definition of collusion (in line with the idea that contracts cannot affect the bargaining procedure):

collusion (cont.)

  - if i arrives before j, i is forced to bring in j's resources as well
  - if j arrives before i, j cannot bring in its resources till i arrives
- in other words, j becomes a dummy-player
- for each coalition S, we can write

$$vC_i^j(S) - v(S) = [vE_i^j(S) - v(S)] + [vI_i^j(S) - v(S)]$$

- if $i \in S$, the first term on the RHS equals 0 because $E_i^j(S) = S$
- if $i \notin S$, the second term on the RHS equals 0 because $I_i^j(S) = S$

collusion (cont.)

- Hence for k's payoffs we find

$$f_k^{\alpha}(vC_i^j) - f_k^{\alpha}(v) = -\sum_{\pi\in\Pi|\pi(j)<\pi(k)<\pi(i)} \alpha_\pi \Delta_{jk}^2 v(\pi^k) + \sum_{\pi\in\Pi|\pi(i)<\pi(k)<\pi(j)} \alpha_\pi \Delta_{jk}^2 v(\pi^k)$$

- recall from above:
  - E_i^j has a negative externality on k if j is complementary to k *in the absence of i*
  - I_i^j has a negative externality on k if j is substitutable to k *in the presence of i*

collusion (cont.)

- assuming that α is symmetric w.r.t. permutations, the "sum" of these effects can be written as

$$f_k^\alpha(vC_i^j) - f_k^\alpha(v) = \sum_{\pi\in\Pi|\pi(j)<\pi(k)<\pi(i)} \alpha_\pi \Delta_{ijk}^3 v(\pi^k)$$

- with "symmetric α" the overall effect of both inclusion and exclusion can be written as

$$\Delta_{jk}^2 v(\pi^k \cup i) - \Delta_{jk}^2 v(\pi^k)$$

- if i increases the complementarity between j and k, the net effect of collusion is to raise k's payoffs
- put differently, if each player k increases the complementarity ($\Delta_{ij}^2 v$) between colluding players i and j, the external effect is positive and collusion is unprofitable

Indisponsable player

- Consider the BW98 set-up with M_a, M_b and R
- we say that R is indispensable to M_b if

$$\Delta_{M_b} v(S) = 0 \text{ if } R \notin S$$

- This implies that M_b and R are complementary because

$$\begin{aligned}\Delta_{M_b R} v(S) &= \Delta_R[\Delta_{M_b} v](S) \\ &= \Delta_{M_b} v(S \cup R) - \Delta_{M_b} v(S \setminus R) = \Delta_{M_b} v(S \cup R) \geq 0\end{aligned}$$

- It follows from equation (80) that an exclusive contract $E^R_{M_a}$ is always profitable
- yet, M_b enters and total surplus is not affected (while the difference between π^c and π^e_a is driving the results in BW98)

Indispensable player (cont.)

- if M_b and R enter in the bargaining ordering before M_a, they cannot generate a surplus until M_a arrives
- M_a then captures this surplus as part of his marginal contribution
- This makes the exclusive contract profitable for M_a
- M_a can compensate R ex ante for accepting such an exclusive contract

Two-sided markets

- Consider a two-sided market with set of players M_x upstream and M_y downstream
- each player $i \in M_x(M_y)$ has endowment $x_i(y_i)$ of factor input
- a coalition S generates value $\phi(X_S, Y_S)$ where $X_S = \sum_{i \in S \cap M_x} x_i, Y_S = \sum_{i \in S \cap M_y} y_i$
- production function satisfies $\phi_x, \phi_y > 0, \phi_{xx}, \phi_{yy} < 0$ and CRS
- Hence we know that

$$\phi_x x + \phi_y y = \phi$$

- Differentiating this w.r.t. y yields $\phi_{xy} \geq 0$
- Hence x and y are complements: an exclusive contract E_i^j between $i, j \in M_x$ is profitable (similarly for $i, j \in M_y$), while an inclusive contract is not

Two-sided markets (cont.)

- effect of exclusive contract between $i \in M_x$ and $j \in M_y$ is not clear because $\phi_{yy} < 0$ while $\phi_{xy} > 0$
- By the end of the day, total surplus is $\phi(\sum_{i \in M_x} x_i, \sum_{i \in M_y} y_i)$ independent of contracts used

Discussion

- this cooperative approach to contracts gives relatively simple conditions to check whether exclusion (inclusion, collusion) contracts are profitable
  - it is not extremely simple since conditions need to be checked for all coalitions (that is for all the different orderings in the bargaining procedure)
- main difference with BW98: (exclusive) contracts here do not affect total payoffs because of equation (79)
- to illustrate, exclusive contract here does not stop other parties from dealing with this agent
- hence an exclusive contract cannot foreclose entry in this set-up
- it only affects the distribution of payoffs
- if competition policy uses a total welfare standard: no reason to forbid exclusive contracts

Part VIII

Summary

Conclusion

- If players can offer menus of contracts, both offer games (no matter what the beliefs are) and bidding games sustain equilibrium outcomes between the pairwise stable and competitive outcomes (SW03,MS03)
- exclusive dealing is an equilibrium outcome if:
  - $x_{-i} = 0$ is contractible and M makes private offers (HT90)
  - in a delegated common agency (bidding) game (MS03)
  - there are contracting externalities in a bidding game or non-coincident markets (BW98)
- these papers analyze the case where there is one side of the market with one player
- extending this to more than 1 player on both sides of the market is non-trivial

Conclusion (cont.)

- cooperative game theory can be used to analyse the effects of contracts in these more complicated situations
- bilateral contracting leads to oversupply compared to efficient outcome if:
  - offer game with private offers, passive beliefs and negative externalities on traders (S99)
  - offer game with public offers and negative externalities on non-traders (S99 (discr.), S03a(non-discr. and incr. extern.))
  - bidding and offer games where menus of contracts are offered (SW03,MS03)
- menu auction where both x_i and x_{-i} are contractible can lead to efficient outcome (BW86)

References

AB87 Aghion, P. and P. Bolton, 1987, "Contracts as a barrier to entry", *American Economic Review*, 77: 388-401

BW86 Bernheim, D. and M. Whinston, 1986, "Menu auctions, resource allocation, and economic influence", *Quarterly Journal of Economics*, 101: 1-31.

BW98 Bernheim, D. and M. Whinston, 1998, "Exclusive dealing", *Journal of Political Economy*, 106: 64-103.

GH80 Grossman, S. and O. Hart, 1980, "Takeover bids, the free rider problem and the theory of the corporation", *Bell Journal of Economics*, 11: 42-64.

References (cont.)

HT90 Hart, O. and J. Tirole, 1990, "Vertical integration and market foreclosure", *Brookings Papers on Economics Activity*, special issue: 205-276.

JMS96 Jehiel, P., B. Moldovanu and E. Stachetti, 1996, "How (not) to sell nuclear weapons", *American Economic Review*, 86: 814-829.

KM98 Kahn, C. and D. Mookerjee, 1998, "Competition and incentives with nonexclusive contracts", *RAND Journal of Economics*, 29 (3): 443-465.

KS86a Katz, M. and C. Shapiro, 1986a, "How to license intangible property", *Quarterly Journal of Economics*, 101: 567-589.

References (cont.)

KS86b Katz, M. and C. Shapiro, 1986b, "Technology adoption in the presence of network externalities", *Journal of Political Economy*, 94: 822-841.

MS03 Martimort, D. and L. Stole, 2003, "Contractual Externalities in Common Agency Games", Advances in Theoretical Economics, 3, n. 1.

RRW91 Rasmusen, E., M. Ramseyer and J. Wiley, 1991, "Naked exclusion", *American Economic Review*, 81: 1137-1145.

S99 Segal, I., 1999, "Contracting with externalities", *Quarterly Journal of Economics*, 114: 337-388.

S03a Segal, I., 2003a, "Coordination and discrimination in contracting with externalities: divide and conquer?", *Journal of Economic Theory*, 113: 147-181.

References (cont.)

S03b Segal, I., 2003b, "Collusion, exclusion, and inclusion in random-order bargaining", *Review of Economic Studies*, 70: 439-460.

SW00 Segal, I., and M. Whinston, 2000, "Naked exclusion: comment", *American Economic Review*, 90: 296-309.

SW03 Segal, I., and M. Whinston, 2003, "Robust predictions for bilateral contracting with externalities", *Econometrica*, 71: 757-791.

W04 Winter, E., 2004, "Incentives and discrimination", *American Economic Review*, 94(3): 764-773.